Chapter 1

Sunday, August 26, 2018 2:27 PM

MA1200 Tutorial Class G Session

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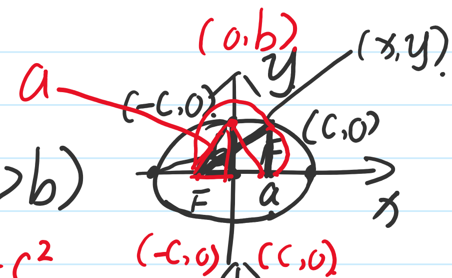
11/09/2018.

TG3 Conic section:

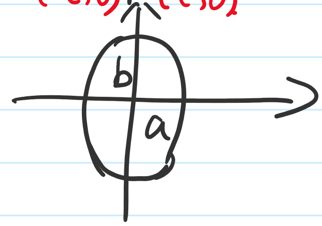
① ellipse: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a>b)$



$a^2=b^2+c^2.$

② parabola:

③ hyperbola:



1. $(x+a)^2+b$ or $(y+a)^2+b$.

(a). $x^2+12x-3$. completing square.

$= x^2+2\times 6x-3$

$(x+y)^2 = x^2+2xy+y^2$.

$= x^2+2\times 6x+6^2-6^2-3$

$= \underline{x^2+2\times 6x+6^2}-6^2-3$ $\qquad \underline{\underline{(x+y)^2}} = x^2+2xy+y^2.$

perfect square.

$= (x+6)^2-36-3$

$= (x+6)^2-39.$

(d) $1y^2+9y+1.$

$= y^2+2\times\frac{9}{2}y+1$

$= y^2+2\times\frac{9}{2}y+(\frac{9}{2})^2-(\frac{9}{2})^2+1.$

$= (y+\frac{9}{2})^2-\frac{81}{4}+1$

$= (y+\frac{9}{2})^2-\frac{77}{4}$

2、 $\underline{(kx+a)^2}+b$ or $\underline{(ky+a)^2+b}$ $= \underline{\underline{k^2y^2}}+2kay+a^2+b.$

(a). $4x^2+24x-9.$

$=(2x)^2+24x-9.$

$= \underline{(2x)^2+2\cdot(2x)\cdot 6+6^2}-6^2-9.$

$= (2x+6)^2-36-9$

$= (2x+6)^2-45.$

(f). $12\sqrt{7}y+\underline{\underline{7y^2}}+21.$

$= (\underline{\underline{\sqrt{7}y}})^2+2\cdot\sqrt{7}y\cdot 6+21$

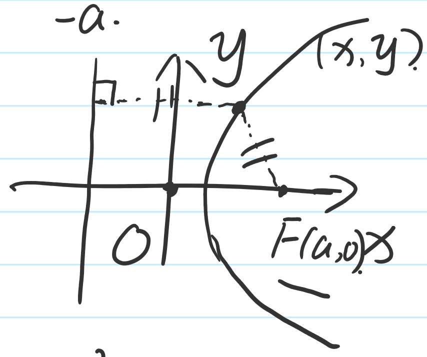
$= \underline{(\sqrt{7}y)^2+2\cdot\sqrt{7}y\cdot 6+6^2}-6^2+21.$

$= (\sqrt{7}y+6)^2-15.$

$= (\sqrt{7}y+6)^2 - 15.$

3. parabola: $\begin{cases} \overset{① y^2}{(y-k)^2} = 4P(x-h) \checkmark \\ \underset{① x^2}{(x-h)^2} = 4P(y-k). \end{cases}$

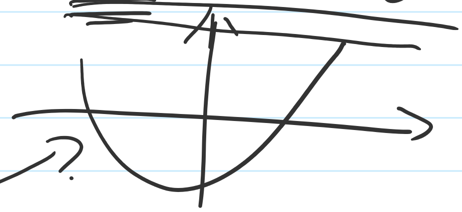
(a). $y^2 - 6y - 24x + 9 = 0.$



$y^2 - 6y + 3^2 - \cancel{3^2} - 24x + \cancel{9} = 0$

$(y-3)^2 = 24x. = 4 \cdot 6(x-0).$

$ax^2 + bx + c = 0$



(f). $4x^2 - 4x - 48y - 47 = 0.$

$(2x)^2 - 4x + 4 - 4 - 48y - 47 = 0$

$(2x-2)^2 - 48y - 51 = 0.$

$4(x-1)^2.$

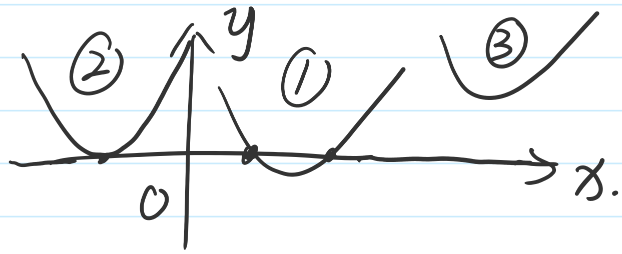
$x^2 - x - 12y - \frac{47}{4} = 0.$

$(x-\frac{1}{2})^2 - \frac{1}{4} - 12y - \frac{47}{4} = 0.$

$(x-\frac{1}{2})^2 = 12y + 12.$

$(x-\frac{1}{2})^2 = 4 \cdot 3(y+1).$

4. $y = x^2+2x-5$. location. curve with x-axis $\Downarrow$ $y=0$.



$$\begin{cases} y = x^2+2x-5. \\ y=0. \end{cases}$$

$$\Rightarrow x^2+2x-5=0.$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$ ✓

$$ax^2+bx+c=0.$$

$a=1$, $b=2$, $c=-5$.

① $\sqrt{b^2-4ac} = \sqrt{4-4\cdot1\cdot(-5)}$
$= \sqrt{24} > 0$.

$$\begin{cases} \text{① } b^2-4ac>0 & \text{two solutions.} \\ \text{② } b^2-4ac=0. & \text{one solution. (two equivalent solutions)} \\ \text{③ } b^2-4ac<0 & \text{no solutions.} \end{cases}$$

$$x = \frac{-2 \pm \sqrt{24}>0}{2\cdot1.}$$
$$= -1 \pm \sqrt{6}.$$

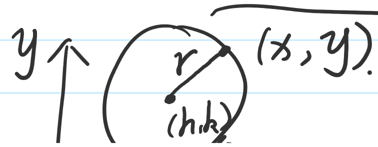
(C). $\begin{cases} y = 3x^2+4x+10. \\ y=0. \end{cases}$ $\Rightarrow 3x^2+4x+10=0.$

① $a=3$, $b=4$, $c=10$

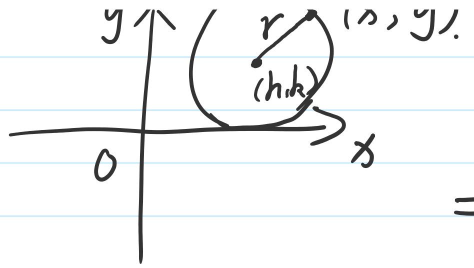
no solution.

$\Rightarrow$ no intersected points. $\sqrt{b^2-4ac} = \sqrt{4^2-4\times3\times10} < 0$

5. Circle $(x-h)^2+(y-k)^2=r^2$.



$\sqrt{(x-h)^2+(y-k)^2}$ = distance



$$\sqrt{(x-h)^2+(y-k)^2} = \text{distance}$$

$(x,y) \to (h,k)$

$= r.$

$\Longrightarrow$

(a). $x^2+y^2-2x+8y+8=0.$

$\Rightarrow x^2-2x + y^2+8y+8=0$

$\Rightarrow (x^2-2x+1)-1+(y^2+8y+4^2)-4^2+8=0.$

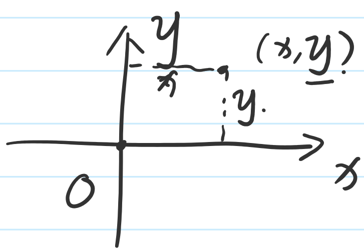
$\Rightarrow (x-1)^2+(y+2)^2-1-16+8=0$

$\Rightarrow (x-1)^2+(y+2)^2 = 9 = 3^2$

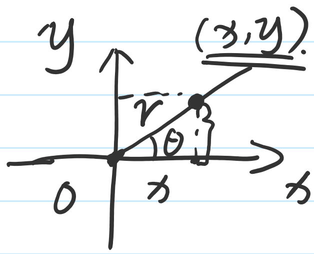
$(1, -2)$

$r=3.$

Polar Coordinates:



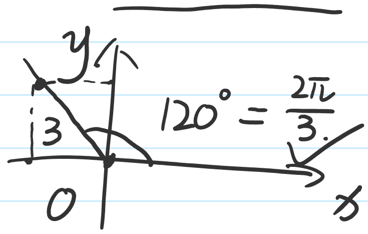
$\longrightarrow$

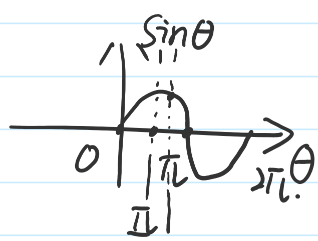


$$\begin{cases} y = r \cdot \sin\theta \\ x = r \cdot \cos\theta. \end{cases}$$

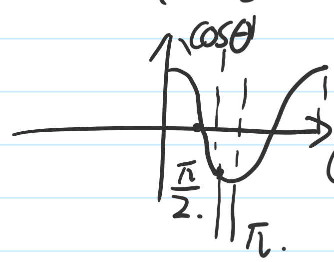
6. $(3, 120°) \Longrightarrow (x, y).$

$x<0$, $y>0.$





$$\begin{cases} y = 3\cdot\sin\frac{2\pi}{3} = 3\cdot\frac{\sqrt3}{2} = \frac{3\sqrt3}{2}. \\ x = 3\cdot\cos\frac{2\pi}{3} = 3\cdot\cos(\pi-\frac{\pi}{3}) = 3\cdot(-\cos\frac{\pi}{3}) \end{cases}$$

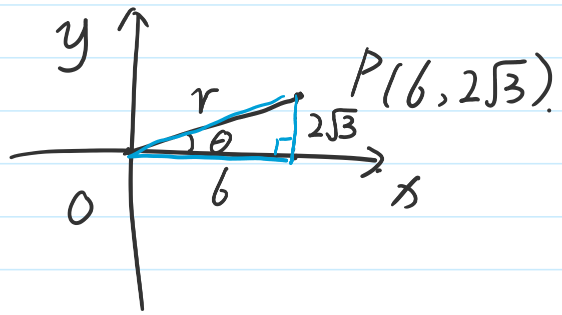


$= 3\cdot(-\frac{1}{2})$

$= -\frac{3}{2}.$

$\frac{1}{2}$. π ... $= -\frac{1}{2}$.

7. $P(6, 2\sqrt{3})$ $\quad -180^\circ < \theta \le 180^\circ \Rightarrow -\pi < \theta \le \pi$.



θ

$\tan\theta = \frac{2\sqrt{3}}{6} = \frac{\sqrt{3}}{3}$

$\theta = \frac{\pi}{6}$.

$r^2 = (2\sqrt{3})^2 + (6)^2$.

$= 48$.

$\therefore r = \sqrt{48} = 4\sqrt{3}$.

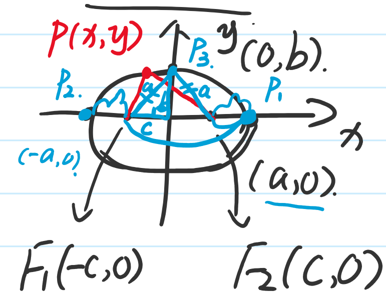
8. $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$, $\quad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$ $\quad (a > b > 0)$.

formula. property. graph.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

$|PF_1| + |PF_2| = 2a$ ✓

$\cancel{2b}$? $\cancel{2c}$?



a, b, c.

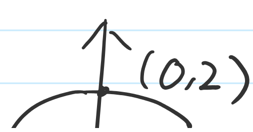
$a^2 = b^2 + c^2$.

(a). $4x^2 + 36y^2 - 144 = 0$.

$4x^2 + 36y^2 = 144$. $\qquad a = 6,\ b = 2$.

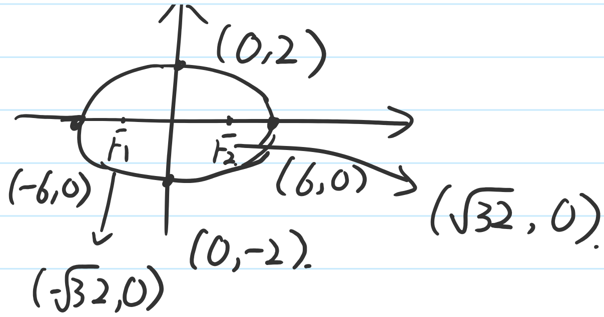
$\frac{4x^2}{144} + \frac{36y^2}{144} = 1$. $\qquad c^2 = a^2 - b^2 = 36 - 4 = 32$.

$\frac{x^2}{36} + \frac{y^2}{4} = 1$



$$\frac{x^2}{36}+\frac{y^2}{4}=1.$$

$$\frac{x^2}{6^2}+\frac{y^2}{2^2}=1.$$



(d). $25x^2+y^2-150x+2y+201=0.$

$\Rightarrow 25(x^2-6x)+(y^2+2y)+201=0.$

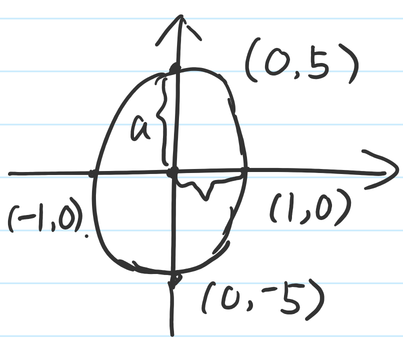
$\Rightarrow 25(x^2-6x+3^2-3^2)+(y^2+2y+1^2-1^2)+201=0.$

$\Rightarrow 25(x-3)^2-225+(y+1)^2-1+201=0$

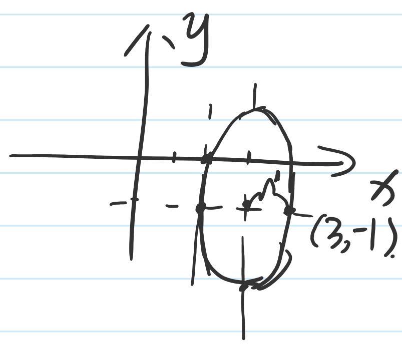
$\Rightarrow 25(x-3)^2+(y+1)^2=25.$

$\Rightarrow \dfrac{(x-3)^2}{1^2}+\dfrac{(y+1)^2}{5^2}=1$

$a=5 \quad b=1. \quad c=\sqrt{a^2-b^2}=\sqrt{5^2-1^2}=\sqrt{24}.$



$$\frac{x^2}{1}+\frac{y^2}{5}=1.$$



center (0,0).

(3,-1).

9. $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ or $\dfrac{(y-k)^2}{a^2}-\dfrac{(x-h)^2}{b^2}=1$

4. $\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$ or $\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$.

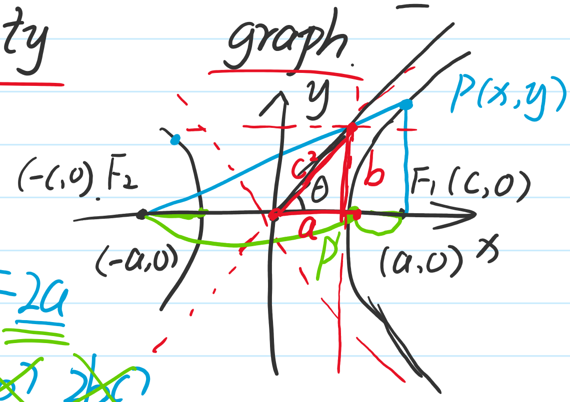
hyperbola. formula. property graph.

$\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$.

$\Big| |PF_2| - |PF_1| \Big| = 2a$

~~2b?~~ ~~2c?~~

$c^2 = a^2 + b^2$.



$y = \tan\theta \cdot x$.

$\tan\theta = \dfrac{b}{a}$.

$\Rightarrow y = \dfrac{b}{a}x$.

or $y = -\dfrac{b}{a}x$.

(a). $16x^2 - 25y^2 + 400 = 0$.

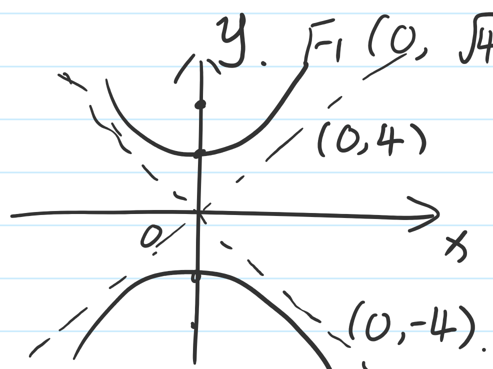
$400 = 25y^2 - 16x^2$.

$\dfrac{25y^2}{400} - \dfrac{16x^2}{400} = 1$.

$\dfrac{y^2}{16} - \dfrac{x^2}{25} = 1$

$\dfrac{y^2}{4^2} - \dfrac{x^2}{5^2} = 1$.

$a = 4$. $b = 5$.



$c^2 = a^2 + b^2 = 4^2 + 5^2 = 41$.

$c = \pm\sqrt{41}$.

(d). $5x^2 - 4y^2 + 10x + 8y + 21 = 0$ ✓

$5(x^2 + 2x + 1 - 1) - 4(y^2 - 2y + 1 - 1) + 21 = 0$.

$$5(x^2+2x+1-1) - 4(y^2-2y+1-1) + 21 = 0$$

$$5(x+1)^2 - 5 - 4(y-1)^2 + 4 + 21 = 0$$

$$5(x+1)^2 - 4(y-1)^2 + 20 = 0$$

$$\frac{(y-1)^2}{5} - \frac{(x+1)^2}{4} = 1 \quad \checkmark$$

$a = \sqrt{5}$

$b = 4$

center $\cancel{(0,0)}$

$(-1, 1)$

10. (a) $5\,\cancel{4}x^2 + 4y^2 - 4x + 12y - 6 = 0$

$$4(x^2 - x) + 4(y^2 + 3y) - 6 = 0$$

$$4\left(x^2 - x + \left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2\right) + 4\left(y^2 + 3y + \left(\tfrac{3}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2\right) - 6 = 0$$

$$4\left(x - \tfrac{1}{2}\right)^2 - 1 + 4\left(y + \tfrac{3}{2}\right)^2 - 9 - 6 = 0$$

$$\cancel{4}\left(x - \tfrac{1}{2}\right)^2 + \cancel{4}\left(y + \tfrac{3}{2}\right)^2 = \cancel{16}\; 4$$

$$\boxed{\left(x - \tfrac{1}{2}\right)^2 + \left(y + \tfrac{3}{2}\right)^2 = 2^2}$$

$\left(\tfrac{1}{2}, -\tfrac{3}{2}\right)$ $\quad r = 2$

11. xy-system

$\downarrow$ rotation θ

$x'y'$-system

$$\begin{cases} x' = \cdots x \\ y' = \cdots y \end{cases}$$

12. (a) $4x^2-9y^2-8x-36y-68=0$

$A=4 \quad C=-9$

$B=0$

$Ax^2+Bxy+Cy^2+Dx+Ey+F=0.$

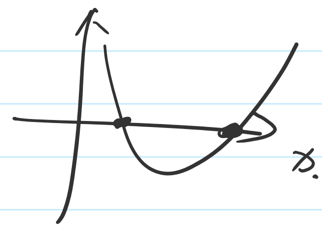
B^2-4AC

$=0^2-4\times4\times(-9)$

>0

$ax^2+bx+c=0.$

$x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}.$



$b^2-4ac:\begin{cases}>0\\=0\\<0\end{cases}$

discriminant.

$B^2-4AC=\begin{cases}<0. & \text{ellipse}\\=0 & \text{parabola.}\\>0 & \text{hyperbola.}\end{cases}$

Chapter 2

Tuesday, October 2, 2018 10:20 AM

Chapter 2 Set function.

1、 $B = \{ x \in \mathbb{R} \mid -11 \le x < -3 \}$ $\mathbb{R}$ infinite

$\{ x \mid x \text{ is real number} \}$

$[-11, -3)$

$C = \{ x \in \mathbb{Z} \mid -11 \le x < -3 \}$ $\mathbb{Z}$

integers finite

~~$[-11, -3)$~~

$\{-11, -10, -9, \cdots, -4\}$.

Set : "$\cup$" "$\cap$" .. $A \cup B$ $\cap$

number 1, 2, 3. "+" "−" "×" "÷"

$X + Y$.

2. $F(x) = 2x - 3, \quad x \in [-1, \infty)$.

$G(x) = x^2, \quad x \in \mathbb{R}$.

$G(x) = x^2, \quad x \in \mathbb{R}.$

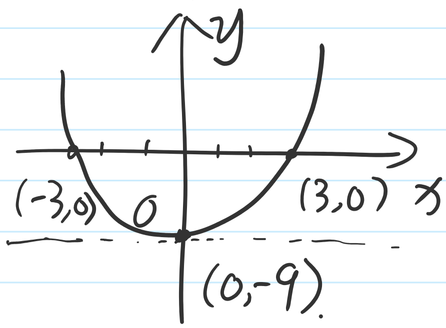
(a)

$F(x) = 2x, \quad x \in \mathbb{R}$ (b)

$G'(x) = x^2, \quad x \in [0, 1]$

3. (a) $y = x^2 - 9$

(i) Graph $\quad y = 4px^2$ $\qquad$ (ii) Analysis



$\begin{cases} y = 0 \\ y = x^2 - 9 \end{cases} \Rightarrow x_1 = 3, \; x_2 = -3.$

$\begin{cases} x = 0 \\ y = x^2 - 9 \end{cases} \Rightarrow y = -9$

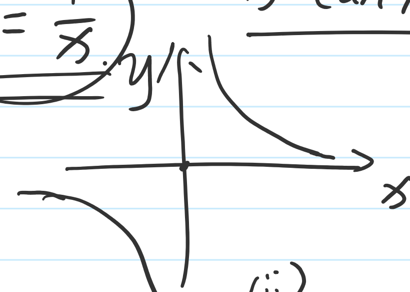
$y = x^2 - 9 \geq 0 - 9$

$y \geq -9$

Domain: $\mathbb{R}$.

Range: $[-9, \infty)$.

(f). $y = \dfrac{5}{x-3}.$ $\qquad y = \dfrac{1}{x}.$ $\qquad$ x can NOT equal to 0

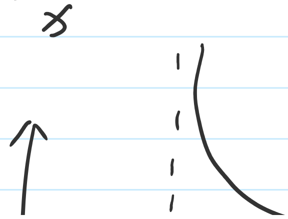


(i).

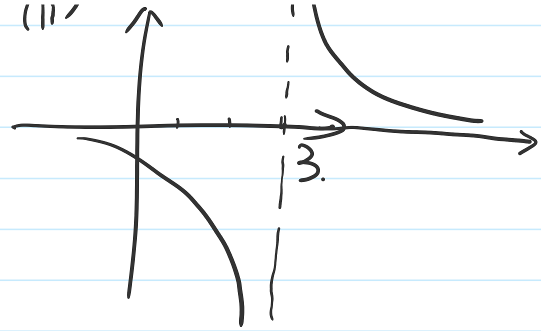
$x - 3 \neq 0 \Rightarrow x \neq 3.$

Domain: $x \in \mathbb{R} \setminus \{3\}$

(ii)



Domain: $x \in \mathbb{R} \setminus \{3\}$

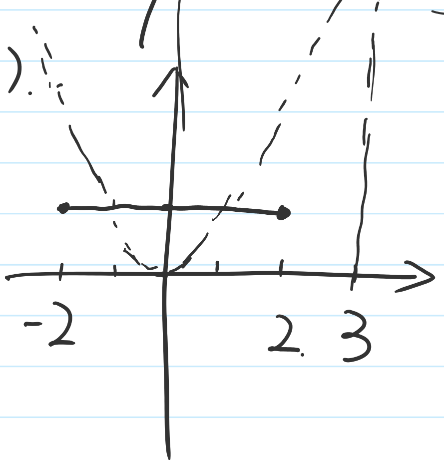


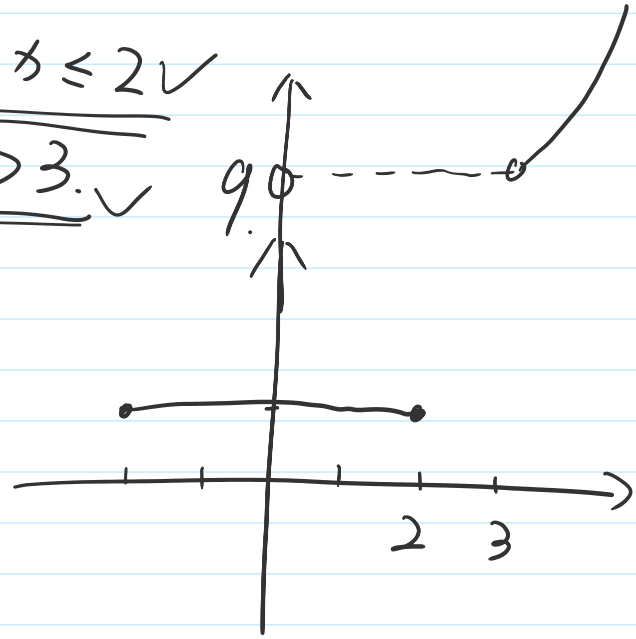
$y \neq 0$.

Range: $y \in \mathbb{R} \setminus \{0\}$

(h). $y = \begin{cases} 1, & -2 \le x \le 2 \\ x^2 & x > 3. \end{cases}$

(i).





Domain: $[-2,2] \cup (3,+\infty)$

$\{x \in \mathbb{R} \mid -2 \le x \le 2$ and $x > 3\}$.

Range: $(-9,+\infty) \cup \{1\}$.

5. $f(x) = x - [x]$ greatest integer which is less or equal to x.

$[1.2] = ?$ 2 $[-1.7] = -2$ -1

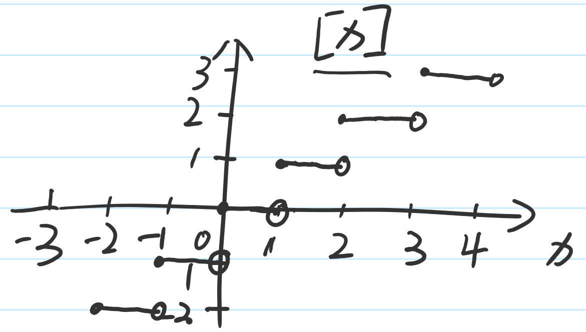
(2)

$-2, -1, 0, 1 \le 1.2$

$-4, -3, -2 \le -1.7$

(-1)

$[2.5] = 2$



$[1.3] = 1$
$[1.81] = 1$

$[1] = 1$

$[2] = 2$

$[3] = 3$

$[-1.1] = -2$

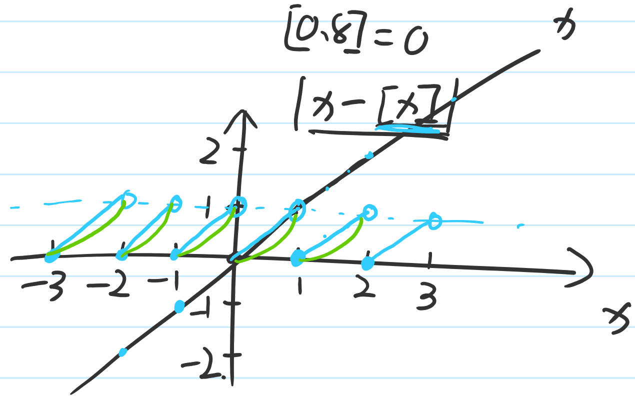
$[-1.2] = -2$

$[-2] = -2$

$[-1] = -1$

$[0.7] = 0$

$[0.8] = 0$



$y = x$

$2 - [2]$

$= 0$

Domain: $\mathbb{R}$

Range: $[0, 1)$

$-(-2)$ $\quad$ $-(-1)$

$+2$ $\quad$ $+1$

$(x - [x])^2$

7. Even: $f(-x) = f(x)$ (1)

Odd: $f(-x) = -f(x)$ (2)

Domain

$f(x) = x^2, \quad x \in \mathbb{R}$

$f(-x) = (-x)^2 = x^2 = f(x)$

$x \in \mathbb{R}$

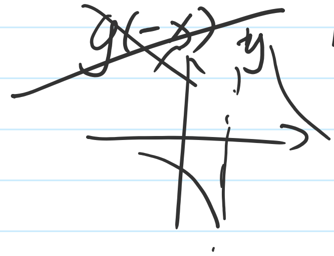
$g(x) = x^2, \quad x \in [0, +\infty)$

(a) $f(x) = \sin(2x) + 5x^3$. $\quad x \in \mathbb{R}$. $\quad \frac{1}{x}$

(a). $f(x) = \sin(2x) + 5x^3$. $x \in \mathbb{R}$. $\left(\frac{1}{x}\right)$

$f(-x) = \sin(2\cdot(-x)) + 5(-x)^3$ $\quad \frac{1}{x-3} \Updownarrow \frac{1}{-x-3}$

$x>0$



$= -\sin 2x - 5x^3$

$= -(\sin 2x + 5x^3)$

$= -f(x)$

odd

9. $f(x) = x^3 + 2$ ($x \in \mathbb{R}$), $\quad g(x) = \frac{2}{x-1}$

$x - 1 \neq 0$

$x \neq 1$

$x \in \mathbb{R}\setminus\{1\}$

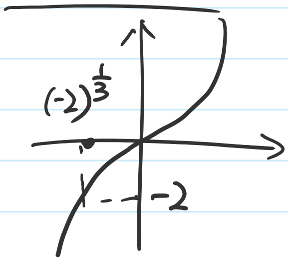
(b). $\left(\frac{g}{f}\right)(x)$

$= \frac{\frac{2}{x-1}}{x^3+2} = \frac{2}{(x-1)(x^3+2)}$

$x-1 \neq 0$ and $x^3+2 \neq 0$.

$x \neq 1$ and $x \neq (-2)^{\frac{1}{3}}$

Domain : set !



$\{x \in \mathbb{R} \mid x \neq 1 \text{ and } x \neq (-2)^{\frac{1}{3}}\}$

$\mathbb{R} \setminus \{1, (-2)^{\frac{1}{3}}\}$

(d). $f\circ g\,(x) = f(g(x)) = (g(x))^3 + 2$.

$\boxed{g(f(x))}$ $= \left(\frac{2}{x-1}\right)^3 + 2$.

$\boxed{g(f(x))} = \left(\frac{2}{x-1}\right)^3 + 2.$

$x \neq 1.$ $\quad \mathbb{R} \setminus \{1\}.$

10. (e). even × odd.

$\Downarrow$ $\quad$ $\Downarrow$

$f(x) = f(-x)$ $\quad$ $g(-x) = -g(x)$

$$f \cdot g(-x) = f(-x) \cdot g(-x) = f(x) \cdot (-g(x)) \overset{?}{=} -f \cdot g(x) \checkmark$$

$$\overset{?}{=} f \cdot g(x)$$

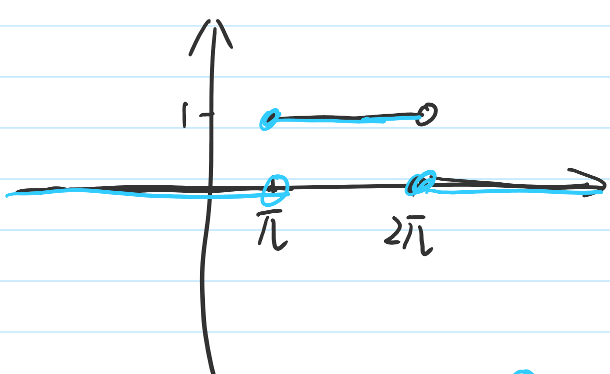
$$\overset{?}{=} \times$$

$\boxed{\text{odd}}$

13. $f(x) = (U_\pi(x) - U_{2\pi}(x))(3 + \sin x).$

$$\checkmark \quad U_a(x) = \begin{cases} 0, & x < a \\ 1, & x \geq a. \end{cases}$$

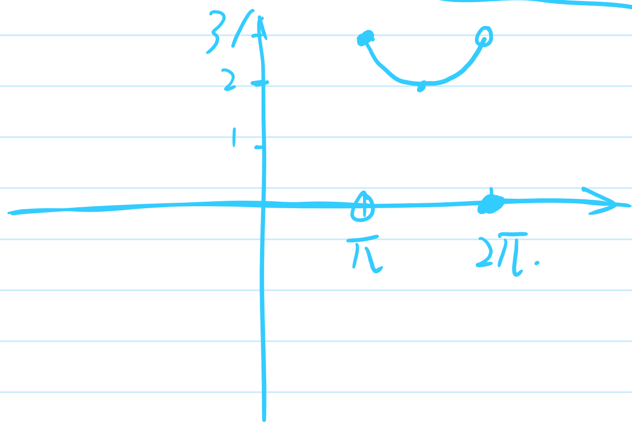
$$U_\pi(x) = \begin{cases} 0, & x < \pi \\ 1, & x \geq \pi. \end{cases} \qquad U_{2\pi}(x) = \begin{cases} 0, & x < 2\pi \\ 1, & x \geq 2\pi. \end{cases}$$

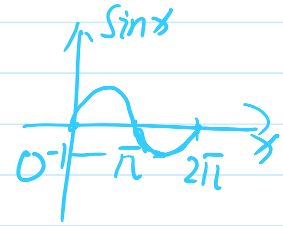


$$U_\pi(x) - U_{2\pi}(x) = \begin{cases} 0-0=0, & x < \pi. \\ 1-0=1, & \pi \leq x < 2\pi. \\ 1-1=0, & x \geq 2\pi. \end{cases}$$

$$(U_\pi(x) - U_{2\pi}(x))(3 + \sin x) = \begin{cases} 0, & x < \pi \end{cases}$$

$$(u_{\pi}(x)-u_{2\pi}(x))(3+\sin x)=\begin{cases}0 & , x<\pi \\ 3+\sin x & , \pi\le x<2\pi \\ 0 & , x\ge 2\pi.\end{cases}$$





Chapter 3.

1. $g(x) = -3x^2 + 24x - 36 = 0$

$ax^2 + bx + c = 0$ $\quad$ $x^2 = 4py$

$x_{1,2}$.

(a). $g(2x)$ $\quad$ $g(-x)$

$g(2x) = -3(2x)^2 + 24(2x) - 36$

$= -12x^2 + 48x - 36$.

$g(-x) = -3(-x)^2 + 24 \cdot (-x) - 36$ $\quad$ $-g(x) = 3x^2 - 24x + 36$

$= -3x^2 - 24x - 36$.

2. (a). $f(x) = 3x^2 + 12x - 36 = 3(x^2 + 4x) - 36$

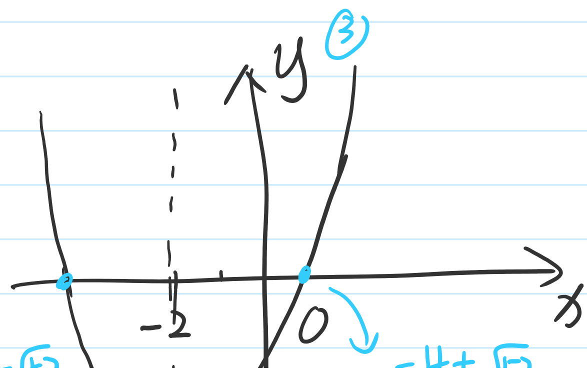
(i). $f(x) = a(x-b)^2 + c$.

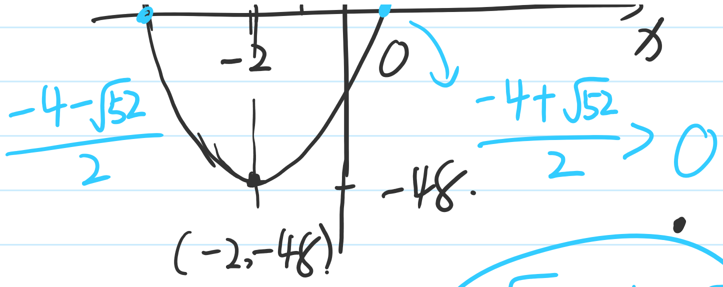
$= 3(x^2 + 4x + 2^2 - 2^2) - 36$

$= 3(x+2)^2 - 12 - 36$

$= 3(x-(-2))^2 - 48$

$a > 0$ $\quad$ $b = -2$





$a > 0$ $b=-2$

$(-2,-48)$

$\sqrt{52} > 4 = \sqrt{16}$

$3x^2+12x-36=0$ $x_{1,2}$

$x^2+4x-9=0$

$a=1,\ b=4,\ c=-9.$

$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$

$b^2-4ac = 4^2 - 4\times 1\times(-9)$

$= 16+36 = 52.$

$x_{1,2} = \frac{-4 \pm \sqrt{52}}{2}$

3.(a). $p(x) = 2x^3+11x^2+3x-4$ $2x+1$

$$
\begin{array}{r}
x^2+5x-1 \\
2x+1 \enclose{longdiv}{2x^3+11x^2+3x-4} \\
\underline{2x^3+x^2} \\
10x^2+3x \\
\underline{10x^2+5x} \\
-2x-4 \\
\underline{-2x-1} \\
-3
\end{array}
$$

$ax-b$

r

5. Factorization:

5. Factorization:

$$P(x) = x^3 + 6x^2 + 3x - 10 = (x-1)(x+2)(x+5)$$

① $x-1$ $\quad$ $x-2$ $\quad$ $x-\frac{1}{2}$ $\qquad$ $ax-b$

$a=1,\ b=-1$

$$x-1 \overline{\smash{)}\,x^3+6x^2+3x-10}^{\;x^2+7x+10}$$

$P\left(\frac{b}{a}\right)=0$ $\qquad$ $P(1) \overset{?}{=} 0$ $\qquad$ $P(2)$

$P(-1) \overset{?}{=} 0$ $\qquad$ $P\left(\frac{1}{2}\right)$

$a_n = 1$

$a_0 = -10$

② $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

$ax-b$ factor $\Rightarrow f\left(\frac{b}{a}\right)=0.$

$\Rightarrow$ b factor of a_0

a factor of a_n.

$a = 1$

$b = 1, 2, 5, 10,$
$-1, -2, -5, -10$

$f\left(\frac{1}{1}\right) = f(1) \overset{?}{=} 0$

$(x-1)$

$f\left(\frac{2}{1}\right) = f(2) \overset{?}{=} 0$

$(x-2)$

6. (e) $h(x) = \dfrac{2x^3+x-5}{(x^3-x^2+2x-2)}$

$= \dfrac{2x^3+x-5}{(x-1)\underbrace{(x^2+2)}_{\geq 0}}$

$P(x) = x^3 - x^2 + 2x - 2 = 0$

$= (x-1)(x^2+2)$

$P(1) = 1^3 - 1^2 + 2\times 1 - 2 = 0$

≥ 0

≥ 2

$\mathbb{R} \setminus \{1\}$

$x^2+2=0$

$P(1) = 1^3 - 1^2 + 2\times 1 - 2 = 0$

$x-1$

$$\begin{array}{r} x^2 + 2 \\ x-1 \,\big)\, x^3 - x^2 + 2x - 2 \\ \underline{\cancel{x^3} - \cancel{x^2}} \\ 2x-2 \\ \underline{2x-2} \\ 0 \end{array}$$

(f). $f(x) = \boxed{\dfrac{(x+3)^2}{x+3}} = \cancel{(x+3)}$

$x+3 \neq 0$

$x \neq -3$ $\quad$ $x \neq -3$

$\boxed{\mathbb{R} \setminus \{-3\}}$

7. partial fractions:

(a). $\dfrac{3x^2+18x+18}{P(x) = x^3+7x^2+14x+8}$.

(i) $P(1) = 1+7+14+8 > 0$

$x-1$

$P(-1) = -1+7-14+8 = 0$

$x-(-1) = x+1$

$$\begin{array}{r} x^2+6x+8 \\ x+1 \,\big)\, x^3+7x^2+14x+8 \end{array}$$

① $\dfrac{f(x)}{(x+a)(x+b)(x+c)} = \dfrac{A}{x+a} + \dfrac{B}{x+b} + \dfrac{C}{x+c}$

② $\dfrac{f(x)}{(x+a)^3}$

③ $\dfrac{f(x)}{(ax^2+bx+c)(x+d)}$

$P(x) = (x+1)(x^2+6x+8)$

$= (x+1)(x+2)(x+4)$

$$\begin{array}{r} x+1 \overline{) x^3+7x^2+14x+8} \\ \underline{x^3+x^2} \\ 6x^2+14x \\ \underline{6x^2+6x} \\ 8x+8 \end{array}$$

$$(a) = \frac{3x^2+18x+18}{x^3+7x^2+14x+8} = \frac{A}{x+1} + \frac{B}{x+2} + \frac{C}{x+4}.$$

$$= \frac{A(x+2)(x+4)+B(x+1)(x+4)+C(x+1)(x+2)}{(x+1)(x+2)(x+2)}$$

$x=-1$ $\quad 3-18+18 = A\cdot(1)\cdot(3)$

$3A = 3$

$A = 1.$

$$= \frac{1}{x+1} + \frac{-3}{x+2} + \frac{-1}{x+4}$$

$x=-2$ $\quad 3\cdot4+18\cdot(-2)+18 = B\cdot(-1)\cdot(2)$

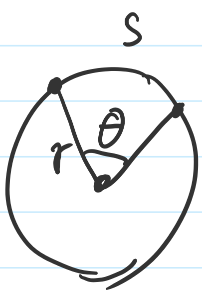
$\Rightarrow B = -3.$

$x=-4$ $\quad 3\cdot(-4)^2-18\cdot4+18 = C\cdot(-3)\cdot(-2)$

$\Rightarrow C = -1.$

Chapter 4:

1.



$\theta = \frac{s}{r}$

$\theta = \pi$, $180°$

$s = \pi \cdot r$

$120° = x$ radians

$\Downarrow$

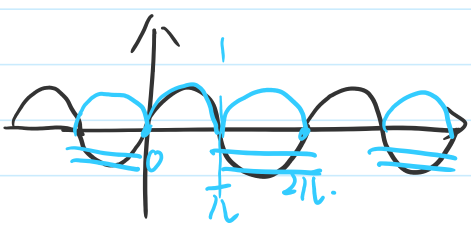
$\frac{2\pi}{3}$ radians

$\frac{\pi \text{ radians}}{180°} = \frac{x \text{ radians}}{120°}$

$x = \frac{120°}{180°} \cdot \pi \cdot \text{radians}$

$= \frac{2}{3}\pi$

2. $f(x) = -2|\sin x|$



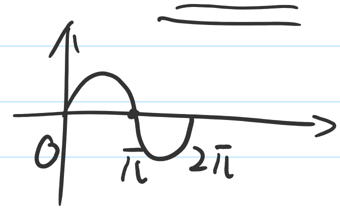
π

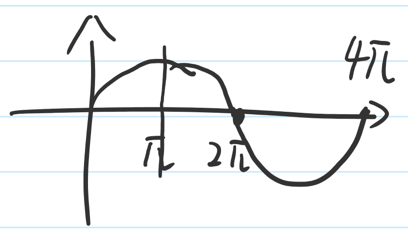
$f(x) = \cos\frac{x}{2}$.

$x = \pi$

$\cos\frac{\pi}{2}$

2π





4π.

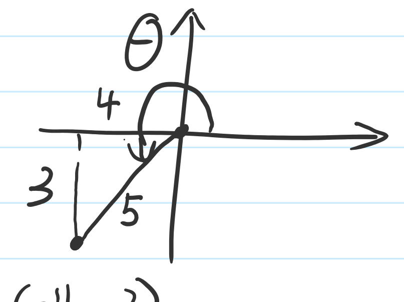
4. (a). $\dfrac{\frac{1}{\cos\theta} - \cos\theta}{\frac{1}{\sin\theta} - \sin\theta} = \tan^3\theta$.

$$\sin\theta - \sin\theta$$

$$= \frac{\frac{1-\cos^2\theta}{\cos\theta}}{\frac{1-\sin^2\theta}{\sin\theta}}$$

$$= \frac{\frac{\sin^2\theta}{\cos\theta}}{\frac{\cos^2\theta}{\sin\theta}} = \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\sin\theta}{\cos^2\theta} = \frac{\sin^3\theta}{\cos^3\theta} = \left(\frac{\sin\theta}{\cos\theta}\right)^3$$

5. $\cos\theta = -\frac{4}{5}$ III $\quad \sin\theta \quad \tan\theta$

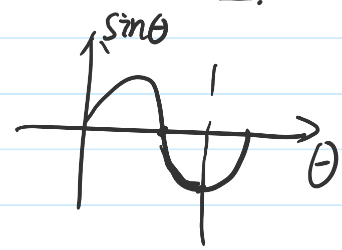


$$\boxed{\sin\theta = \frac{-3}{5}}$$

$$\tan\theta = \frac{-3}{-4} = \frac{3}{4}$$

III.

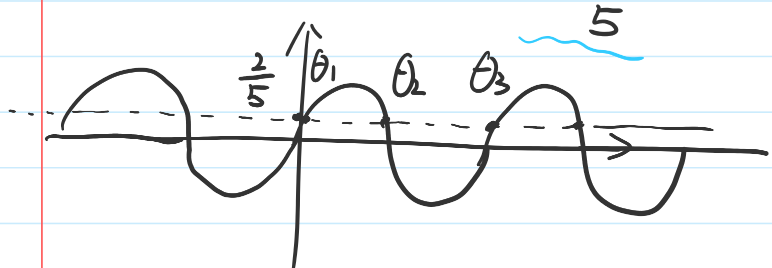
$\pi < \theta < \frac{3\pi}{2}$.



7. $\sin\left(\sin^{-1}\frac{2}{5}\right) = \sin(\theta_1, \theta_2, \theta_3)$ $\quad \sin^{-1}x = \theta \Rightarrow \sin\theta = x$

$= \frac{2}{5}$

$$\frac{1}{\sin x} = (\sin x)^{-1}$$



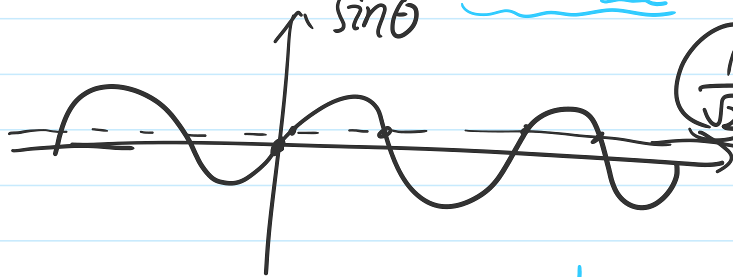
$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin^{-1}\left(\sin\frac{\pi}{4}\right) = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4} + 2n\pi, \ \frac{3\pi}{4} + 2n\pi,$$

$n \in \mathbb{Z}$

$\uparrow \sin\theta$

$n \in \mathbb{Z}.$



$\frac{1}{\sqrt{2}}$

$\frac{\pi}{4}$

Principal range:

$\sin^{-1} x : -\frac{\pi}{2} \le y \le \frac{\pi}{2}.$ ✓

$\cos^{-1} x : 0 \le y \le \pi$

$\tan^{-1} x : -\frac{\pi}{2} < y < \frac{\pi}{2}.$

8. (b) $\tan 165^\circ = \tan(135^\circ + 30^\circ) = \frac{\tan 135^\circ + \tan 30^\circ}{1 - \tan 135^\circ \cdot \tan 30^\circ}$

$45^\circ, 30^\circ, 60^\circ, 90^\circ$ $\qquad \pi - \frac{\pi}{4} \qquad \frac{\pi}{6}$

$\frac{\pi}{4}$ $\qquad = \frac{3\pi}{4}$

$= \frac{-1 + \frac{1}{\sqrt{3}}}{1 - (-1) \times \frac{1}{\sqrt{3}}} = \frac{1 - \sqrt{3}}{1 + \sqrt{3}}.$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

$= \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3}) \cdot (1-\sqrt{3})}$

$\tan \frac{3\pi}{4} = -\tan \frac{\pi}{4} = -1$

11. $4 \cos A \cos\left(\frac{2\pi}{3} + A\right) \cos\left(\frac{2\pi}{3} - A\right) = \cos 3A$

$= \cancel{4} \cos A \cdot \cancel{\frac{1}{2}} \left\{ \cos\left(\frac{2\pi}{3} \cancel{+A} + \frac{2\pi}{3} \cancel{-A}\right) + \cos\left(\cancel{\frac{2\pi}{3}} + A - \cancel{\frac{2\pi}{3}} + A\right)\right.$

product → sum : $\cos A \cdot \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$= 2\cos A \cdot \{\cos\left(\frac{4\pi}{3}\right) + \cos 2A\}$ $\qquad \cos \frac{4\pi}{3} = \ldots \frac{\pi}{}$

$$= 2\cos A\cdot\left\{\cos\left(\frac{4\pi}{3}\right) + \cos 2A\right\}$$

$$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$$

$$= 2\cdot\cos A\left\{-\frac{1}{2} + \cos 2A\right\}$$

$$= -\cos A + 2\cos A\cdot\cos 2A.$$

$$= -\cos A + 2\times\frac{1}{2}\left\{\cos 3A + \cos(-A)\right\}$$

$$= \cancel{-\cos A} + \cos 3A + \underbrace{\cos(-A)}_{\cancel{\cos A}} = \cos 3A$$

13. (b) $2\sin^2 x + \sin x - 1 = 0.$

$$2x^2 + x - 1 = 0$$

$$x_{1,2} = \frac{-b\pm\sqrt{b^2-4ac}}{2a}.$$

$y=\sin x$

$$\Longrightarrow 2y^2 + y - 1 = 0.$$

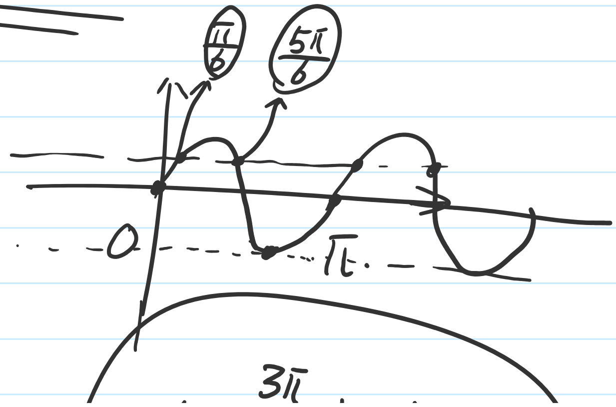
$$\Longrightarrow (2y-1)(y+1) = 0$$

$$\Longrightarrow y_1 = \frac{1}{2},\quad y_2 = -1.$$

$$\Longrightarrow \sin x = \frac{1}{2},\ \text{or}\ \sin x = -1.$$

$$\boxed{x = \frac{\pi}{6} + 2n\pi,\ \frac{5\pi}{6} + 2n\pi.}$$

$$n = \cdots -1, 0, 1, 2, \cdots$$



$$\Longrightarrow \left(x = (-1)^n\cdot\frac{\pi}{\ } + n\pi\right)$$

$$\Rightarrow \boxed{x = (-1)^n \cdot \frac{\pi}{6} + n\pi}$$

$$x = \frac{3\pi}{2} + 2n\pi$$

$n = \ldots -1, 0, 1, 2 \ldots$

Chapter 5

Tuesday, October 23, 2018 10:27 AM

Chapter 5.

$f(x) = e^x$

$= 5^x$

$f(x) = \ln x$

$= \log_e x$

$= \log_{10} x$

1. (ii). $a > b > 1$.

$\Rightarrow \log_a 10 > \log_b 10$. ? X F

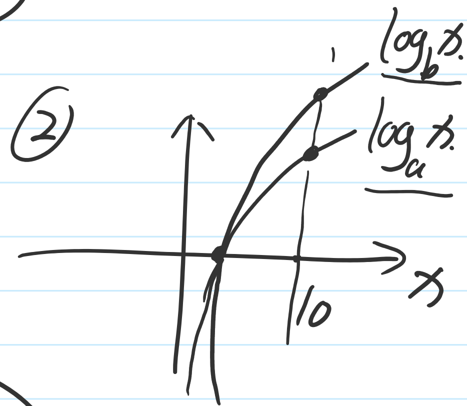
① $a = 100$, $b = 10$

$\log_{100} 10 = \log_{100} 100^{\frac{1}{2}} = \frac{1}{2} \log_{100} 100 = \frac{1}{2}$.

$\log_{10} 10 = 1$.

$\log_b 10 > \log_a 10$

②

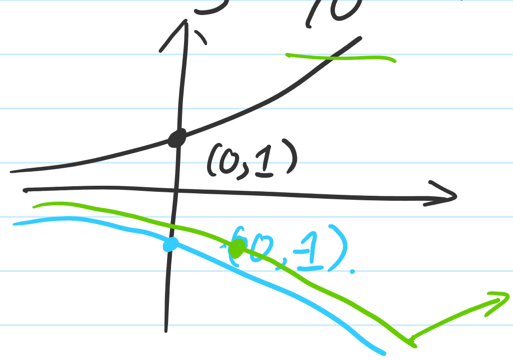


4. (iii) $f(x) = -10^{(x-1)}$

$10^x \to -10^x \to -10^{x-1}$

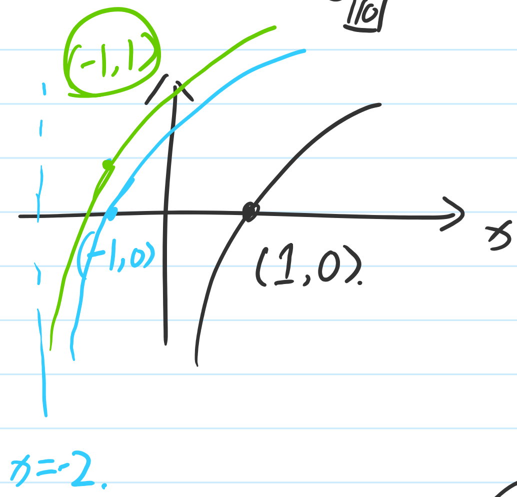
$\Downarrow$

$(1, -1)$



(iv). $f(x) = 1 + \log_{1/10}(x+2)$

(iv). $f(x) = 1 + \log_{\boxed{10}}(x+2)$



$\log x \longrightarrow \log(x+2) \rightarrow 1+\log(x+2)$

$x=1$ $\quad$ $x=-1$.

$\log 1=0$ $\quad$ $\log 1=0$.

$(-1,0)$

-2
-5

5. (a). $y = \log\frac{10}{x^2} \to x \neq 0 \to \mathbb{R}\backslash\{0\}$

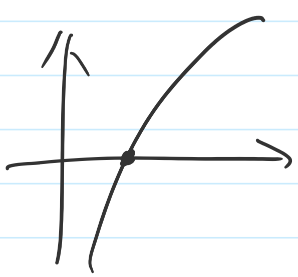
$\begin{cases} \log_a(M \cdot N) = \log_a M + \log_a N \\ \log \frac{M}{N} = \log M - \log N. \end{cases}$

$= \log_{\boxed{10}} 10 - \log x^{②}$

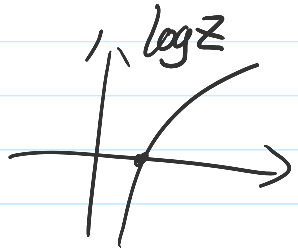
$= 1 - 2\log x \to x>0$

Domain: $\mathbb{R}\backslash\{0\}$

Range: $\mathbb{R}$.



$z = \frac{10}{x^2} > 0$



$\log_a x = \dfrac{\log_b x}{\log_b a}$.

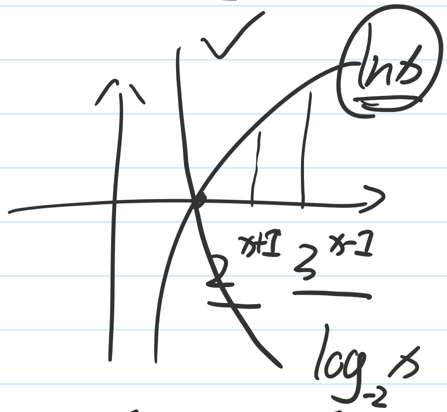
e $\quad$ ln10 $\quad$ ln11

6. $2^x = 3 \Rightarrow x = \log_2 3 = \frac{\ln 3}{\ln 2} = ?$

6. $2^{x} = 3 \Rightarrow x = \underbrace{\log_2 3}_{} = \dfrac{\ln 3}{\ln 2} = ?$

$3^{x-1} = 2^{x+1} \Rightarrow \ln 3^{x-1} = \ln 2^{x+1}$

Remark: Do ln both H.S equation



$(x-1) \cdot \ln 3 = (x+1) \ln 2$

$\Downarrow$

$x \cdot \ln 3 - \ln 3 = x \cdot \ln 2 + \ln 2.$

$\Downarrow$

$x(\ln 3 - \ln 2) = \ln 3 + \ln 2$

$\Downarrow$

$x = \dfrac{\ln 3 + \ln 2}{\ln 3 - \ln 2} = ?$

$3^{x-1} > 2^{x+1}$

$\ln 3^{x-1} > \ln 2^{x+1}$

$\Downarrow$

$x > \cdots$

7. $\ln(y-5) = kx + c.$ $\qquad y = ?(x)$

$e \qquad e$

$\Downarrow$

$e^{\ln(y-5)} = e^{kx+c}.$

$\Downarrow$

$y - 5 = e^{kx+c}$

$y = e^{kx+c} + 5.$

$e^{\ln y} = y$

$5^{\log_5 x} = x$