Assignment 1

Monday, September 9, 2019 5:03 PM

MA2172 Tutorial Class A session.

Tutor: QI kunlun

E-mail: kunlun.qi@my.cityu.edu.hk

Office: Room 1392, FYW Building

(I) Chapter 4, 5, 6, 7 basic probability, distribution..

(II) Chapter 8, 9, 10 inferential statistic: test, hypothesis..

(III) Chapter 13 statistical analysis: linear correlation, regression..

Chapter 4. Assignment 1.

$\boxed{1}$. (a) $\boxed{1}$ $\boxed{2}$ $\boxed{3}$

Sample space: (1, 2), (1, 3), (2, 3)

$\downarrow$ 3 $\quad$ $\downarrow$ $\boxed{4}$ $\quad$ $\downarrow$ 5

$\uparrow$ $\quad$ $\uparrow$ $\quad$ $\uparrow$

(2, 1), (3, 1), (3, 2)

$$P = \frac{2}{6} = \frac{1}{3}$$

$\boxed{(b)}$ $\underline{P(A) = 2P(B)}$ $\underline{P(B) = 2P(C)}$

$\Rightarrow P(A) = 4P(C)$ ✓

As $P(A) + P(B) + P(C) = 1$.

$\Rightarrow 4P(C) + 2P(C) + P(C) = 1$

$\Rightarrow 7P(C) = 1$

$\Rightarrow \underline{P(C) = \frac{1}{7}}$ $\Rightarrow \underline{P(A) = \frac{4}{7}}$, $\underline{P(B) = \frac{2}{7}}$.

$\boxed{2.}$ Alex Bill Chen. 1st $P(A) = \frac{1}{2}$

$P(B) = (1-\frac{1}{2}) \cdot \frac{1}{2}$

$\underline{1^{st} \text{ round}}$ $\frac{1}{2}$ $(1-\frac{1}{2}) \cdot \frac{1}{2}$ $(1-\frac{1}{2})(1-\frac{1}{2}) \cdot \frac{1}{2}$ $P(C)$

(a). $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{8}$.

(b) 2 Rounds

$\underline{2^{nd} \text{ round}}$. $(1-\frac{1}{2})^3 \cdot \frac{1}{2} = \frac{1}{16}$ $(1-\frac{1}{2})^4 \cdot \frac{1}{2} = \frac{1}{32}$ $(1-\frac{1}{2})^5 \cdot \frac{1}{2} = \frac{1}{64}$

$P(A) = \frac{1}{2} + \frac{1}{16} = \frac{9}{16}$ $P(B) = \frac{1}{4} + \frac{1}{32} = \frac{9}{32}$ $P(C) = \frac{1}{8} + \frac{1}{64} = \frac{9}{64}$.

(C). continue until someone win.

$\boxed{n \to +\infty}$ $\frac{1}{2}, \frac{1}{2^4}, \cdots$

$P(A) = \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \cdots = \frac{1}{2}(1 + \frac{1}{2^3} + \frac{1}{2^6} + \cdots)$ $\frac{1}{8}$

$$P(A) = \frac{1}{2} + \frac{1}{2^4} + \frac{1}{2^7} + \cdots = \frac{1}{2}\left(1 + \frac{1}{8} + \frac{1}{8^2} + \cdots\right)$$

$N \to +\infty$ $2, 2^4, \cdots$ $\frac{1}{8}$

$$= \frac{1}{2} \cdot \frac{1\left(1 - \left(\frac{1}{8}\right)^N\right)}{1 - \frac{1}{8}} \quad \left(\frac{1}{8}\right)^N \xrightarrow{N\to\infty} 0$$

$$= \frac{1}{2} \cdot \frac{1 \cdot 1}{1 - \frac{1}{8}} = \frac{4}{7}$$

$$\boxed{a_n = a_1 \cdot r^{n-1}}$$

$a_1 \; a_2 \; \cdots \; a_n.$ $\qquad \frac{a_n}{a_{n-1}} = r$

$$\sum_{n=1}^{N} a_n = \frac{a_1(1 - r^N)}{1 - r} = S_n$$

$$P(B) = \frac{1}{2^2} + \frac{1}{2^5} + \frac{1}{2^8} + \cdots = \frac{1}{4}\left(1 + \frac{1}{8} + \frac{1}{8^2} + \cdots\right)$$

$$P(C) = \frac{1}{2^3} + \frac{1}{2^6} + \frac{1}{2^9} + \cdots$$

$$= \frac{1}{8}\left(1 + \frac{1}{8} + \frac{1}{8^2} + \cdots\right)$$

$$= \frac{1}{8} \cdot \frac{1}{1 - \frac{1}{8}} = \frac{1}{7}.$$

$$= \frac{1}{4} \cdot \frac{1\left(1 - \left(\frac{1}{8}\right)^N\right)}{1 - \frac{1}{8}} \quad \left(\frac{1}{8}\right)^N \to 0 \quad N \to \infty$$

$$= \frac{1}{4} \cdot \frac{1}{1 - \frac{1}{8}} = \frac{2}{7}.$$

$$P(A) + P(B) + P(C) = 1 \; ? \Rightarrow \frac{4}{7} + \frac{2}{7} + \frac{1}{7} = 1$$

(II). $P(A) = 2P(B) \qquad P(B) = 2P(C)$

by 1(b) $\Rightarrow$ $P(A) = \frac{4}{7}, \; P(B) = \frac{2}{7}, \; P(C) = \frac{1}{7}$

3. In past: 1 $\quad P(C) = P$

3\. In past: 1 $\quad P(C) = P$

Now: 3 $\begin{cases} ① & P \\ ② & P \\ ③ & P \end{cases} \Rightarrow P(C) = ?$

$P(C) = P($correct decision by $\boxed{\text{majority rule}})$

$= P($at least 2 of 3 make correct decision$)$

$= P($all 3 correct$) + P($2 of 3 correct$)$

$= \boxed{P^3 + 3P^2(1-P)} \Rightarrow \binom{n}{r} P^r (1-P)^{n-r}$

${}_nC_r = \binom{n}{r} = C_n^r = \quad \begin{cases} r=3 \\ r=2 \end{cases}$

(b) $P = 0.1$. $\quad P(C) = 0.028$

(c) $P = 0.8$. $\quad P(C) = 0.896$.

(d). when now $\geq$ past. $\Rightarrow P = ?$

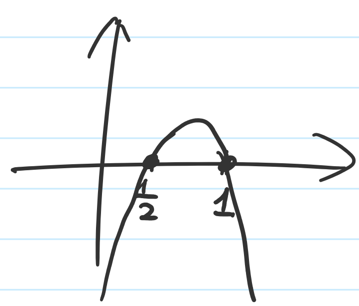
$P^3 + 3P^2(1-P) \geq P$

$P^3 + 3P^2(1-P) - P \geq 0$.

$P^2 + 3P(1-P) - 1 \geq 0$.

$-2P^2 + 3P - 1 \geq 0$

$(-2P+1)(P-1) \geq 0$



$(-2P+1)(P-1) \geq 0$ $\Rightarrow \frac{1}{2} \leq P \leq 1.$

$P_1 = \frac{1}{2}, \; P_2 = 1.$

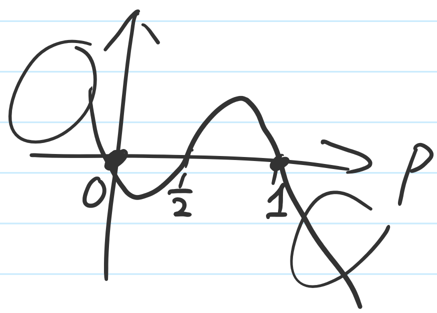
(c) now = past.

$P_0^3 + 3P^2(1-P) = P \Rightarrow P_1 = \frac{1}{2}, \; P_2 = 1.$

$P_3 = 0$

$P(-2P+1)(P-1) = 0$

$P_1 = \frac{1}{2}, \; P_2 = 1, \; P_3 = 0$



5. 25 < 4 defective, 21 Non-defective. "3"

(a) P(all 3 are defective) $\boxed{25}$

$= \frac{4}{25} \times \frac{3}{24} \times \frac{2}{23} = 0.00174.$

(b) P(exactly two are defective)

	case I	case II	III
1st	D	D	N
2nd	D	N	D
3rd	N	D	D

$= \frac{4}{25} \times \frac{3}{24} \times \frac{21}{23} + \frac{4}{25} \times \frac{21}{24} \times \frac{3}{23}$

(I) $+ \frac{21}{25} \times \frac{4}{24} \times \frac{3}{23} = 3 \times \frac{3 \times 4 \times 21}{25 \times 24 \times 23} = 0.0548.$

$$\equiv \frac{\cdots}{25\times24\times23} = 0.0548.$$

(II) $$\frac{\binom{4}{2}\cdot\binom{21}{1}}{\binom{25}{3} = {}_{25}C_3} = \frac{\frac{4\times3}{2\times1}\cdot 21}{\frac{25\times24\times23}{3\times2\times1}} = \frac{3\times\cancel{2\times1}\times\frac{4\times3}{\cancel{2\times1}}\times21}{25\times24\times23} = 0.0548.$$

$${}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

(c) P(Neither is defective)

$$= \frac{21}{25}\times\frac{20}{24}\times\frac{19}{23} = 0.578.$$

6.

| Inspector |

$P(D) = 0.3$

30% Defective (D)
- 0.9 Removed (R) $P(R|D)$
- 0.1 NOT-Removed ($\bar{R}$) $P(\bar{R}|D)$

70% Non-Defective ($\bar{D}$)
- 0.2 Removed (R) $P(R|\bar{D})$
- 0.8 NOT-Removed ($\bar{R}$) $P(\bar{R}|\bar{D})$

$P(\bar{D}) = 0.7$

(a). $$P(D|R) = \frac{P(D\cap R)}{P(R)} = \frac{P(R|D)\cdot P(D)}{P(R|D)\cdot P(D) + P(R|\bar{D})\cdot P(\bar{D})}$$

$$P(R|D) = \frac{P(R\cap D)}{P(D)} \Rightarrow P(D\cap R) = P(R|D)\cdot P(D)$$
$$= 0.9\times0.3$$
$$= 0.27$$

Bayes's Rule

$$P(R) = P(R\cap D) + P(R\cap\bar{D})$$

Re → D, Re – $\bar{D}$

$$= P(R|D)\cdot P(D) + P(R|\bar{D})\cdot P(\bar{D})$$
$$= 0.9\times0.3 + 0.2\times0.7$$

$$\frac{0.9\times0.3}{0.9\times0.3+0.2\times0.7} = 0.6585.$$

$$P(R\cap D) = \cancel{P(D|R)}\cdot\cancel{P(R)}$$

$P(R\cap D) = \cancel{P(D|R)\cdot P(R)}$

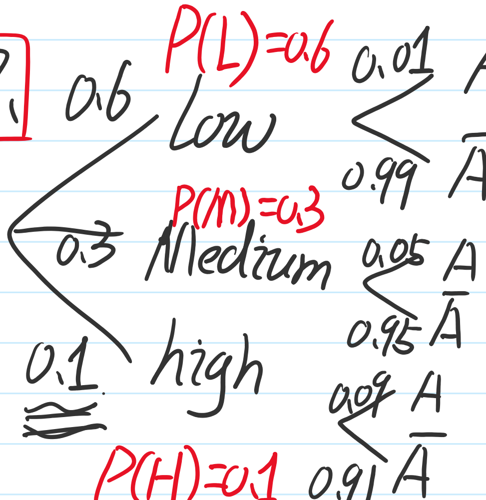
(b). $P(D|\bar{R}) = \dfrac{P(D\cap\bar{R})}{P(\bar{R})} = \dfrac{P(\bar{R}|D)\cdot P(D)}{1-P(R)}$

$\boxed{P(\bar{R})+P(R)=1}$

$= \dfrac{0.1\times 0.3}{1-(0.9\times 0.3+0.2\times 0.7)}$

$= \dfrac{0.03}{0.59} = \dfrac{3}{59} \approx 5\%$

$\boxed{7.}$



$P(A|L)=0.01$

$P(\bar{A}|L)=0.99$ ✓

$P(A|M)=0.05$

$P(\bar{A}|M)=0.95$ ✓

$P(A|H)=0.09$

$P(\bar{A}|H)=0.91$ ✓

(a) $P(A) = P(A\cap L) + P(A\cap M) + P(A\cap H)$

$= P(A|L)\cdot P(L) + P(A|M)\cdot P(M) + P(A|H)\cdot P(H)$

$= 0.6\times 0.01 + 0.3\times 0.05 + 0.1\times 0.09$

$= 0.03.$

(b). $P(H|A) = \dfrac{P(H\cap A)}{P(A)} = \dfrac{0.1\times 0.09}{0.03} = 0.3$

$$\underline{\underline{P(A)}} = \frac{\quad}{0.03} = \underline{\underline{0.3}}$$

(c). $P(\bar{H} | A) = 1 - 0.3 = 0.7$

A A

$\boxed{1}$ $\boxed{2}$

0.7 $\bar{H}|A$, 0.3   0.7 $\bar{H}|A$, 0.3

(I) $P(\text{At least one is High} | A)$

$= 1 - P(\text{None of them is High} | A)$

$= 1 - 0.7 \times 0.7 = 0.51$

(II) $P(\text{At least one High} | A)$

$= 0.3 \times 0.3 + 0.3 \times 0.7 + 0.7 \times 0.3 = 0.51$.

$\boxed{8.}$

	Gold	Silver	Bronze	Total
United States	39	25	33	97
Russia	32	28	28	88
China	28	16	15	59
Australia	16	25	17	58
Others	186	205	235	626
Total	301	299	328	928

(i) $P(\text{G} | \text{China}) = \dfrac{28}{28+16+15} = 0.4746$

(ii) $P(\text{China} | \text{G}) = \dfrac{28}{39+32+28+16+186} = 0.093$

(iii) $P(\text{G}) = \dfrac{301}{928} = 0.3244$

$P(\text{G} | \text{US}) = \dfrac{39}{39+25+33} = 0.4021$

$$P(\mathrm{G} \mid \mathrm{US}) = \frac{39}{39+25+33} = 0.4021$$

$\because P(\mathrm{G}) \neq P(\mathrm{G} \mid \mathrm{US})$

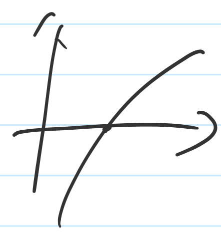
The two events are not independent.

(iv)

	Gold	Silver or Bronze	Total
US or Russia	71	114	185

$$P(\text{No gold medal for the winners} \mid \text{US or Russia}) = \frac{114}{185} \times \frac{113}{184} = 0.3784$$

9. (ii) $1 - 0.9^N \geq 0.3$



$0.9^N \leq 0.7$

$\log 0.9^N \leq \log 0.7$

$N \geq \cdots$

$N \cdot \log 0.9 \leq \log 0.7$

$N = 4$

$N \geq \dfrac{\log 0.7}{\log 0.9} = 3.\cdots$

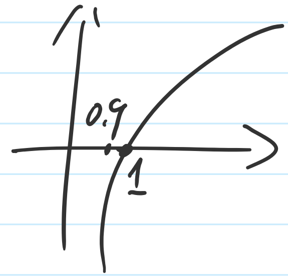
(b) "n" P("at least" one of n defective")

$1 - P(\text{None of them is defective.})$

$= 1-(0.9)^{n} \geq 0.3$

$\Rightarrow \quad 0.9^{n} \leq 0.7$

$\Rightarrow \quad \log 0.9^{n} \leq \log 0.7$



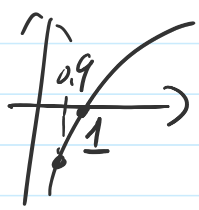
$\Rightarrow \quad n \underline{\log 0.9} \leq \log 0.7$

$\Rightarrow \quad n \boxed{\geq} \dfrac{\log 0.7}{\log 0.9} \approx 3. \ldots$

$\underline{\log 0.9} < 0$

$\Rightarrow$ " $\underline{n} = 4$ "

$0.9^{N} \leq 0.7$



$\log 0.9^{N} \leq \log 0.7$

$N \cdot \underline{\log 0.9} \leq \underline{\log 0.7}$

$N \geq \dfrac{\log 0.7}{\log 0.9}$

$3. \ldots$

N integer

$N = 4$

Assignment 2

Tuesday, September 24, 2019 8:57 PM

1.

"X"	1	2	3	4	5	6
"f(x)"	50	50	50	20	-10	-40
P	0.52	0.26	0.12	0.05	0.03	0.02
g(x)	70	70	70	70-A	70-2A	70-3A

$\boxed{70-59.41}$

30

"A"

59.41

Mean of "f(x)"

$$E[f(x)] = 50\times0.52 + 50\times0.26 + 50\times0.12 + 20\times0.05 + (-10)\times0.03 + (-40)\times0.02$$

$$= 44.9$$

$\boxed{44.9 + 15 = 59.9}$

$$E[g(x)] = 70\times0.52 + 70\times0.26 + 70\times0.12 + (70-A)\times0.05 + (70-2A)\times0.03 + (70-3A)\times0.02 = 59.9$$

$$\Rightarrow A = 59.41.$$

2. A B C $\bar{A}$

Peter Paul Mary

Peter Paul Mary

$P(A)=0.25 \quad P(B)=0.3 \quad P(C)=0.4$

① $P(B|A)=0 \Rightarrow P(\bar{B}|A)=1.$

② "$P(C|A)$" $=0.8$ ✓ $\Rightarrow \dfrac{P(A\cap C)}{P(A)}$

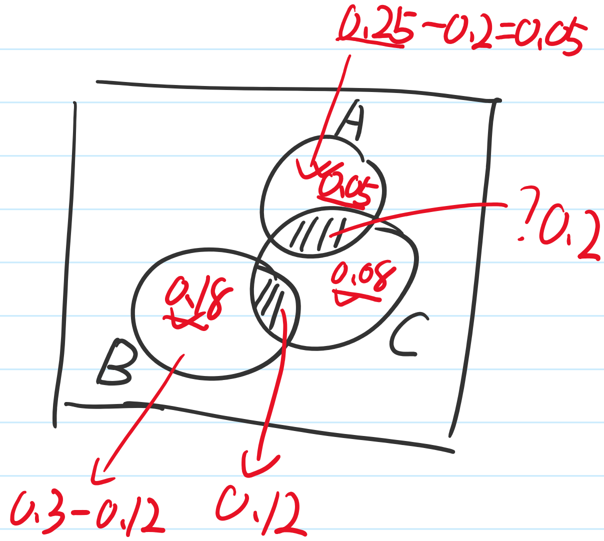
$P(A\cap C)$

$=P(C)\cdot P(A|C)$

$=P(A)\cdot P(C|A)$ ✓

③ $P(B\cap C)=P(B)\times P(C)=0.3\times 0.4=0.12.$

x	0	1	2	~~3~~
P	0.37	0.31	0.32	0



$P(A\cap C)=P(A)\cdot P(C|A)$

$=0.25\times 0.8$

$=0.2.$

(2) Mean $:= E(x) = 0\times 0.37+1\times 0.31+2\times 0.32$

$=0.95$

$SD := \sqrt{\sum_{x=0}^{2} x^2P(x)-\mu^2}$

"$\mu=\sum xP(x)$" $=0.8292.$

$\sum P(x)=1$

$\sigma^2=\sum[(x-\mu)^2P(x)]$

$=\sum[(x^2-2\mu x+\mu^2)P(x)]$

$=\sum[x^2P(x)]-2\sum[\mu xP(x)]+\sum[\mu^2P(x)]$

$=\sum[x^2P(x)]-2\mu\sum xP(x)$

$+\mu^2\sum P(x)$

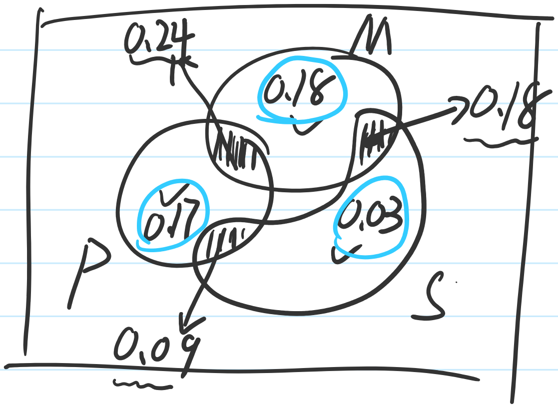
$$\sum P(x) = 1$$

$$- 2\mu \sum xP(x) + \mu^2 \sum P(x) \quad (=1)$$

$$= \sum [x^2 P(x)] - 2\mu^2 + \mu^2$$

$$= \sum [x^2 P(x)] - \mu^2$$

3. $P(M) = 0.6$, $P(P) = 0.5$, $P(S) = 0.3$



① $P(P \mid M) = 0.4 = \dfrac{P(P \cap M)}{P(M)}$

$\Rightarrow P(P \cap M) = 0.4 \times 0.6 = 0.24$

② $P(M \cap S) = P(M) \cdot P(S) = 0.6 \times 0.3 = 0.18$.

③ $P(P \cap S) = \frac{1}{2} P(M \cap S) = \dfrac{0.18}{2} = 0.09$

x	0	1	2	~~3~~
P	0.11	0.38	0.51	0

(2) Mean $:= E(x) = 0 \times 0.11 + 1 \times 0.38 + 2 \times 0.51 = 1.4$

SD $:= \sqrt{\sum x^2 P(x) - \mu^2}$

$= \sqrt{1^2 \times 0.38 + 2^2 \times 0.51 - 1.4^2}$

$= 0.6782$

67.82%

$= 0.6782$

(3) $P(M \cap S \mid \bar{P}) = \dfrac{P(M \cap S \cap \bar{P})}{P(\bar{P})}$

$= \dfrac{P(M \cap S)}{1 - P(P)}$

$= \dfrac{0.18}{0.5} = 0.36$ #

4. "4"

0.9 Non-

0.1 Defective

Y	0	1	2	3	4
C	0	6	22	48	84
P	0.6561	0.2916	0.0486	0.0036	0.0001

$P(Y=0) = \binom{4}{0}(0.1)^0(0.9)^4 = 0.6561$

$P(Y=1) = \binom{4}{1}(0.1)^1 \cdot (0.9)^3 = 0.2916$

$P(Y=2) = \binom{4}{2}(0.1)^2 \cdot (0.9)^2 = 0.0486$

$P(Y=3) = \binom{4}{3}(0.1)^3 \cdot (0.9)^1 = 0.0036$

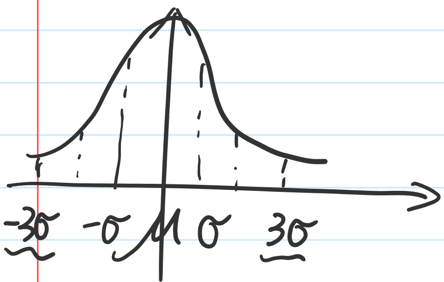
$$P(Y=4) = \binom{4}{4}(0.1)^4 \cdot (0.9)^0 = 0.0001.$$

(b) $$E[C] = \sum_{Y=0}^{4} P(Y)[C = 5Y^2 + Y] = 3$$

Assignment 3

Tuesday, October 1, 2019 7:50 PM

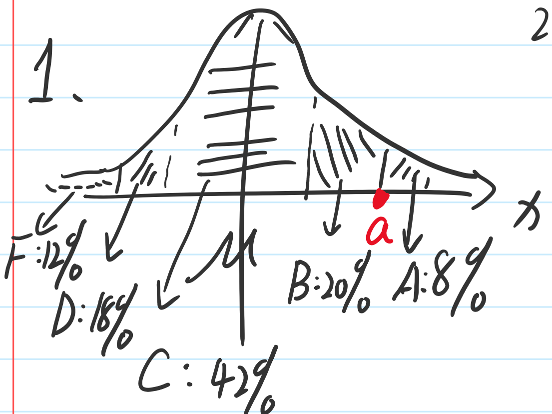
Chapter 6 "Normal distribution"



"3σ" Rule 99.6..%

$28+42=70$

1\.



a: Obtain A.

exceed a to obtain A.

$$z=\frac{x-\mu}{\sigma}$$

$$P(X\geq z_{\Delta})=8\%$$

$z=1.41$ $x\geq 1.41$

$$\Rightarrow \frac{a-\mu}{\sigma}=1.41$$

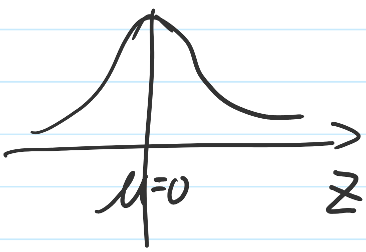
$\mu=72$

$$\Rightarrow a=(12.5\times 1.41)+72$$

$\sigma=12.5$

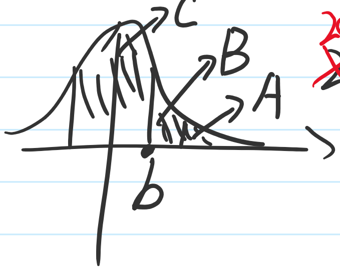
$$\Rightarrow a=89.5.$$

$\mu=0$, $\sigma=1$.



$$z=\frac{x-\mu}{\sigma}$$

b: better than C.



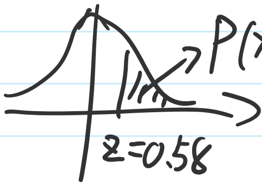
~~20~~ 28%

$$z=\frac{b-\mu}{\sigma}$$

$$P(X\geq z_{\Delta})=\cancel{20}\,28\%$$

$$z=0.58$$

$\rightarrow P(X\geq z)=\cancel{20}\,28\%$ $\Rightarrow \frac{b-\mu}{\ }=0.58$

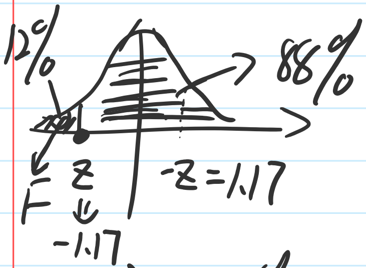


$P(x \geq z) = \overset{28}{\cancel{20}}\% \Rightarrow \dfrac{b-\mu}{\sigma} = 0.58$

$z = 0.58$

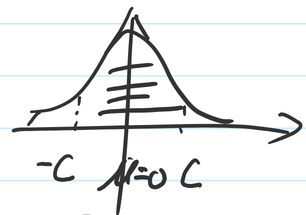
$\xRightarrow[\sigma=12.5]{\mu=72} b = (0.58 \times 12.5) + 72 = \underline{79.2}$

C: Pass the course : D or better.



$P(x \geq z) = 88\%$

$P(x \geq -1.17) = 88\%$



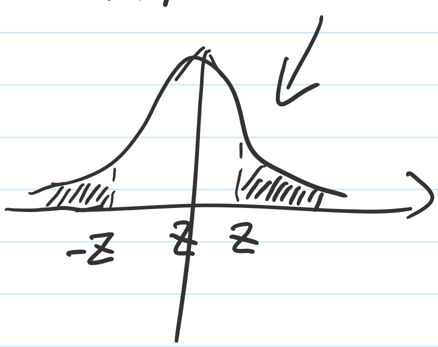
$\boxed{P(-c) + P(c) = 1}$

$\underline{P(x \geq -z) = 12\%} \Rightarrow \dfrac{c-\mu}{\sigma} = -1.17$

$-z = 1.17$

$\Rightarrow z = -1.17$

$\xRightarrow[\sigma=12.5]{\mu=72} c = 57.3.$ #



2. $P(x < \underline{116}) = 0.2$. $\quad P(x < 328) = 0.9$.

1st step

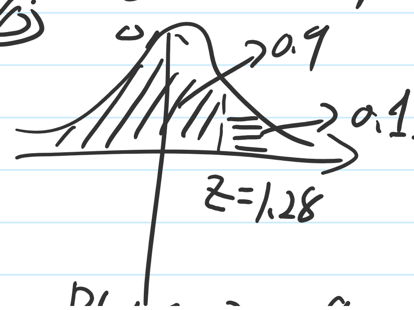
$P\left(\dfrac{x-\mu}{\sigma} < \dfrac{116-\mu}{\sigma}\right) = \underline{0.2}$ $\quad P\left(\dfrac{x-\mu}{\sigma} < \dfrac{328-\mu}{\sigma}\right) = 0.9$

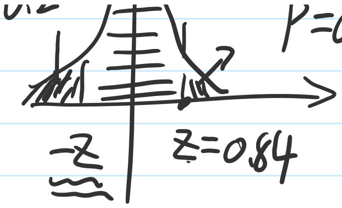
< 0.5

z

$P = 0.2$

$P(x \geq z) = 0.2$





$P(x \ge z) = 0.2$

$z = 0.84$

$\Rightarrow -z = -0.84$

$P(x \le z) = 0.9$

$P(x \ge z) = 0.1$

$z = 1.28$

$P\left(\frac{x-\mu}{\sigma} > \frac{116-\mu}{\sigma}\right)$

$\Rightarrow \frac{116-\mu}{\sigma} = -0.84$ ✓ $\quad \Rightarrow \frac{328-\mu}{\sigma} = 1.28$ ✓

$$\begin{cases} \frac{116-\mu}{\sigma} = -0.84 \\ \frac{328-\mu}{\sigma} = 1.28 \end{cases} \Rightarrow \begin{aligned} 116 &= -0.84\sigma + \mu \quad (1) \\ 328 &= 1.28\sigma + \mu \quad (2) \end{aligned} \Rightarrow$$

$(1) - (2)$

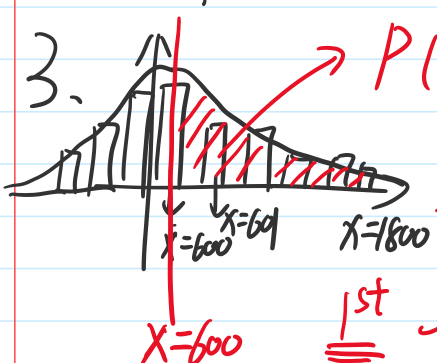
$\sigma = 99.06$ ✓

≈ 100 ✓

$\mu = 201.2$ ✓

≈ 200 ✓

3.



$P(x \ge 601) = P(x \ge 600.5)$

$P(x > 600) = P\left(\frac{x-\mu}{\sigma} > \frac{600.5-\mu}{\sigma}\right)$

2nd continuity correction.

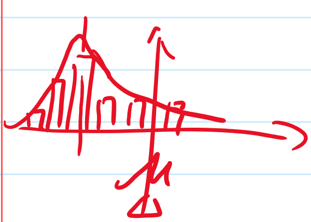
$n = 1800$

$P = 0.3$

1st $\mu = nP = 540 \ge$ "5"

$\sigma = \sqrt{nP(1-P)} = 19.44$

$\mu < 5$

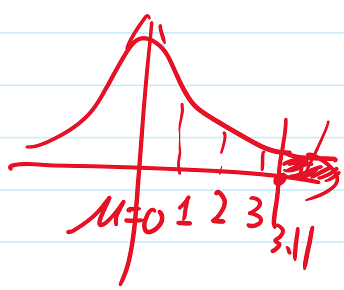


$= P\left(\frac{x-\mu}{\sigma} > \frac{600.5-540}{19.44}\right)$ ✓

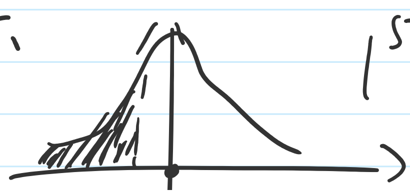
$= P(z > 3.11)$ ✓ $\quad ?3\sigma$

3.1

$= 0.0009.$ #



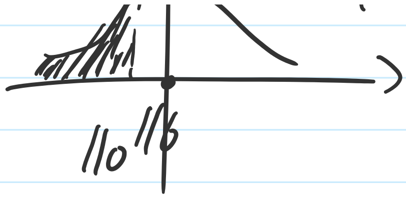
4.

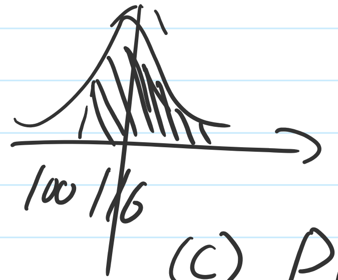


$= nP$

1st: $\mu = 200 \times 0.58 = 116 > 5$

$\sigma = nP(1-P) = 1.96$





$\sigma = np(1-p) = 6.98$

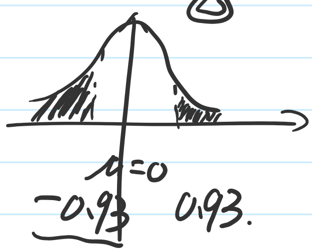
(b)

$P(X < 110)$

$= P(X < 109.5)$

$= P\left(\frac{X-\mu}{\sigma} < \frac{\cancel{110}\ 109.5-\mu}{\sigma}\right)$

$= P(Z < -0.93)$



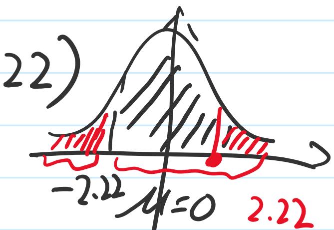
$= P(Z \geq 0.93) = 0.1762.$

(c) $P(X \geq 100)$

$= P(X > 100.5)$

$= P\left(\frac{X-\mu}{\sigma} > \frac{100.5-\mu}{\sigma}\right)$

$= P(Z \geq -2.22)$

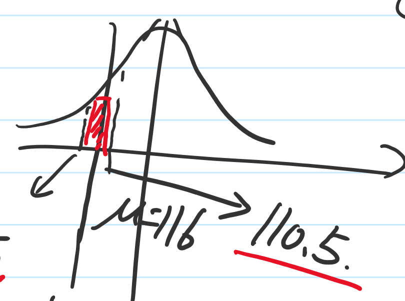


$= 1 - 0.01321$

$= 0.9868.$ $P(Z \geq 2.22) = 0.01321$

(a) $P(X=110) = \binom{200}{110}(0.58)^{110}(0.42)^{90} = 0 \times 10^{-27}$?

$P(X=110) = P(109.5 < X < 110.5)$



$= P\left(\frac{109.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{110.5-\mu}{\sigma}\right)$ 110

$= P(-0.93 < Z < -0.79)$

$= 0.3238 - 0.2852$

$= 0.0386$

5 (a) $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$n = 100$

$p = 0.05$

$$= \binom{100}{0}(0.05)^0(0.95)^{100} + \binom{100}{1}(0.05)^1(0.95)^{99}$$

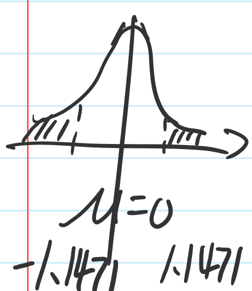
$$+ \binom{100}{2}(0.05)^2(0.95)^{98}$$

$$= 0.11826 \checkmark$$

(b) $\mu = np = \underline{\underline{5}}$

$\sigma = \sqrt{np(1-p)} = 4.75$

(c) $P(X \le 2) = P(X < 2.5) = P\left(\frac{x-\mu}{\sigma} < \frac{2.5-5}{\sqrt{4.75}}\right)$



$$= P(z < -1.1471)$$

$$= P(z > 1.1471)$$

$$= 0.1260 \checkmark$$

(d) 1^{st}: $\mu = np \ge$ "5"

$= 5$

As $np = 5$ (NOT > 5). $nq = 95$, $n > 20$.

It's NOT very symmetric distribution.

6. $P(x \ge 50 \mid x < 62) = \dfrac{P(50 \le x < 62)}{P(x < 62)}$ ✓

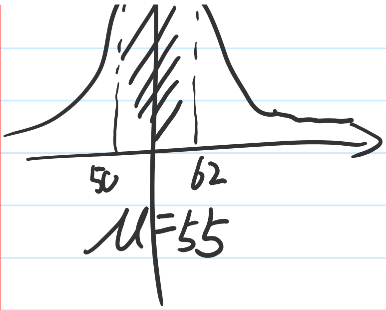
$n = 100$ ✓

$p = 0.55$

$= P(49.5 < x < 61.5)$

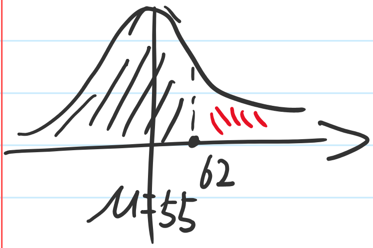
$\mu = np = 55$

$\sigma = \sqrt{npq}$



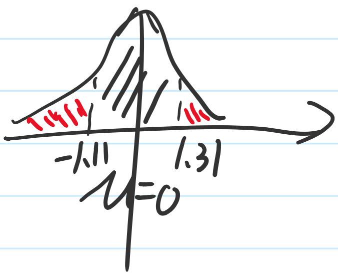
$$= \frac{P(49.5 < x < 61.5)}{P(x < 61.5)}$$

$\sigma = \sqrt{npq} = 4.9749.$



$$= \frac{P\left(\frac{49.5-55}{4.9749} < z < \frac{61.5-55}{4.9749}\right)}{P\left(x < \frac{61.5-55}{4.9749}\right)}$$

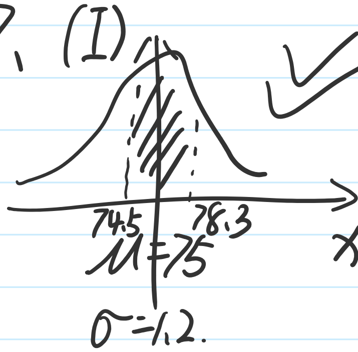
$$= \frac{P(-1.11 < z < 1.31)}{P(z < 1.31)}$$



$$= \frac{1-0.0951-0.1335}{1-0.0951}$$

$= 0.8525.$

7. (I)



$74.5 \le X \le 78.3.$

$$\boxed{76.35-1.95 \le X \le 76.35+1.95}$$

x = diameter

"P": for each one screw, what's probability satisfy requirement of company B.

||

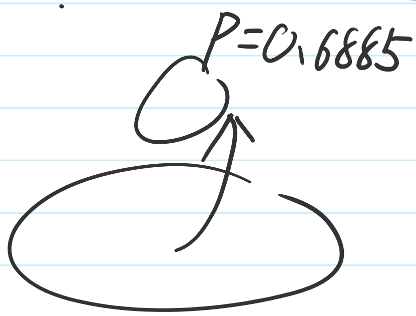
0.6885.

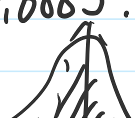
$P(74.4 < X < 78.3)$

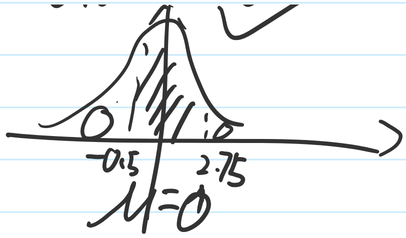
$$= P\left(\frac{74.4-75}{1.2} < z < \frac{78.3-75}{1.2}\right).$$

$= P(-0.5 < z < 2.75).$

$= 0.6885.$







"45 out of n" satisfy my requirement.

(II)

Y: number of screws satisfied.

$$P(Y \geq 45) = P(Y=45) + P(Y=46) \cdots P(Y=n)$$

"n"

$p = 0.6885$

$q = 1 - 0.6885$

$\sigma = \sqrt{npq} = \sqrt{0.2145n}$.

$\mu = np = 0.6885n$

$$= P(X > 44.5)$$

$$= P\left(\frac{X-\mu}{\sigma} > \frac{44.5 - 0.6885n}{\sqrt{0.2145n}}\right) \geq 0.98.$$

$P(z^*) = 0.02$

$$\frac{44.5 - 0.6885n}{\sqrt{0.2145n}} < -2.055.$$

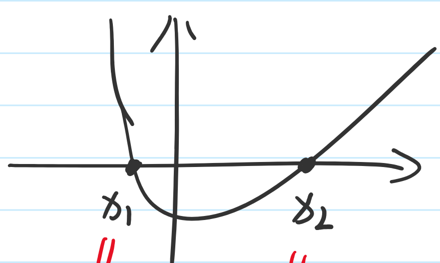
$z^* = -2.055$

$$\Rightarrow 44.5 - 0.6885n < -0.9518\sqrt{n}$$

$$\Rightarrow 0.6885n - 0.9518\sqrt{n} - 44.5 > 0.$$

$\sqrt{n} = x$ $\qquad = 0$

$$0.6885x^2 - 0.9518x - 44.5 = 0 \checkmark$$



$\Rightarrow x > x_2$ or $x < x_1$

$$ax^2 + bx + c = 0$$

$$\boxed{x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.} \Rightarrow \begin{array}{l} x_1 = -7.3779 \\ x_2 = 8.7603 \end{array}$$

$$\boxed{x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}} \Rightarrow \begin{array}{l} x_1 = -7.3779 \\ x_2 = 8.7603. \end{array}$$

$$\Rightarrow \cancel{\sqrt{n} < -7.3779} \text{ or } \boxed{\sqrt{n} > 8.7603.}$$

(rejection)

minimum.

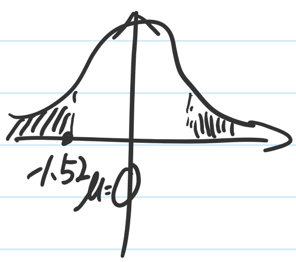
$$\Rightarrow n > 76.74 \Rightarrow n = 77. \ \#$$

Assignment 4

Wednesday, October 16, 2019 11:00 AM

1. $\underline{\underline{X}}$: diameter. $\mu = 2.63$ ✓ $\sigma = 0.25$ ✓

(a) $P(\underline{\underline{X < 2.25}}) = P\left(\frac{X-\mu}{\sigma} < \frac{2.25-2.63}{0.25}\right)$

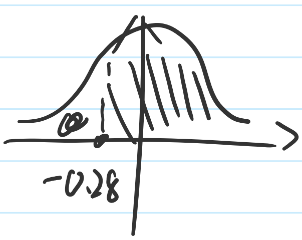


$= P(Z < -1.52)$

$= P(Z > 1.52)$

$= 0.0643 = 6.43\%$

(b) $P(\underline{\underline{X > 2.56}}) = P\left(\frac{X-\mu}{\sigma} > \frac{2.56-2.63}{0.25}\right)$



$= P(Z > \underline{-0.28})$

$= 1 - \underline{P(Z > 0.28)}$

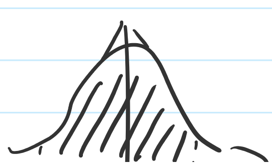
$= 0.6103 = 61.03\%$

(c) $\underline{\underline{100}}$ sample $n = 100$. "$\underline{\underline{CLT}}$" Information about $\underline{\text{sample}}$ $\underline{\text{distribution of}}$

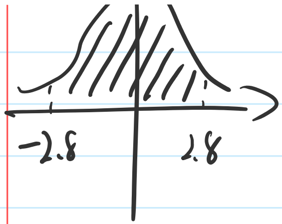
$\underline{\underline{\mu_{\bar{X}}}} = \mu = 2.63$, $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{0.25}{\sqrt{100}} = 0.025$. $\underline{\text{sample}}$ $\underline{\underline{\text{mean}}}$.

$P(\bar{X} > \underline{2.56}) = P\left(\frac{\bar{X}-\mu_{\bar{X}}}{\sigma_{\bar{X}}} > \frac{2.56-2.63}{0.025}\right)$ "$\underline{\underline{\mu_{\bar{X}}}}$" as long as $\underline{\underline{n}}^{=100}$ is big enough.

$= P(Z > -2.8)$



$= 1 - P(Z > 2.8)$



$= 1 - P(z > 2.8)$

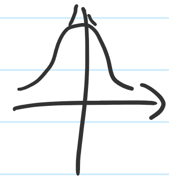
$= 0.9974$

big enough.

$\boxed{\text{"1, 2, 3, 4"}}$

$n = 2$ $\quad\quad \bar{x} = \mu_{\bar{x}}$

"(1,1)	1
(1,2)	1.5
(2,3)	⋮
⋮	⋮
(4,4).	



2(a) Let Σx represent the total weight for this sample.

$$P\left(\sum_{i=1}^{25} x > 4000\right) = P\left(\text{"}\bar{x}\text{"} > \frac{4000}{25}\right)$$

$$= P(\bar{x} > 160)$$

$$= P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{160 - 300}{10}\right)$$

$$= P(z > \underset{\smile}{-14}.0)$$

$$\approx 100\%$$

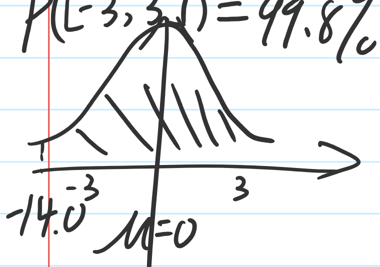
$\sigma^2 = 2500 \rightarrow \sigma = 50$

$\mu_{\bar{x}} = \mu = 300$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{50}{5} = 10$

"3σ" $\quad \sigma = 1$

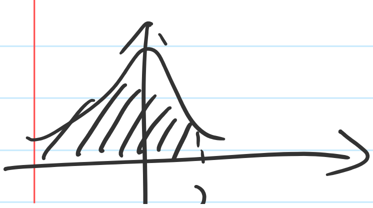
$P([-3, 3]) = 99.8\%$



(b) $P(\Sigma x < 8000) = P(\bar{x} < 320)$

$$= P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} < \frac{320 - 300}{10}\right)$$

$$= P(z < \overset{2\sigma}{2.00})$$



$= P(z < 2.00)$

$= 0.9772$

3. (a). Step I: set up.

(i) parameter of concern: Mean compressive strength of cement --- "$\bar{x}$"

(ii) H_0: $\bar{x} \geq 5000$

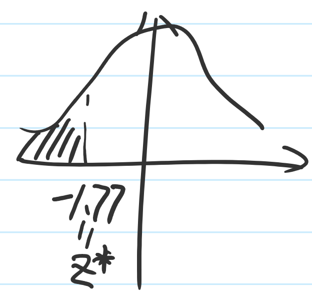
H_a: $\bar{x} < 5000$ ✓

Step II: test statistic z with σ known.

level of significance: $\alpha = 0.02$.

$n = 50$, $\sigma = 120$, $\sigma_{\bar{x}} = \frac{120}{\sqrt{50}}$

$$z^* = \frac{\text{"}4920\text{"} - 5000}{\frac{120}{\sqrt{50}}} = -1.77.$$



$P(z < z^*) = P(z > 1.77) = 3.84\%$

$> \alpha = 2\%$

Step III:

(i) Decision: fail to reject H_0

(ii) Conclusion: At the level of 2% significance there is not enough evidence to conclude that mean compressive strength conclude that mean compressive strength is less than 5000.

< 5000

Chapter 8

(I) P-value Method ✓

(II) Classical Method. △

(III). Estimation of Mean μ. (σ know). ✓

$$\left[\bar{x} - z\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}} \, , \, \bar{x} + z\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}} \right].$$

(2) $95\% = 1-\alpha \Rightarrow \alpha = 0.05$

(i) confidence coefficient: $z\left(\frac{\alpha}{2}\right) = z(0.025) = 1.96$.

(ii) Error: $E = z\left(\frac{\alpha}{2}\right) \cdot \frac{\sigma}{\sqrt{n}} = 1.96 \cdot \frac{120}{\sqrt{50}} = 33.3.$

(iii) Confidence interval: $\bar{x} - E = 4970 - 33.3 = 4936.7$

$\bar{x} + E = 4970 + 33.3 = 5003.3.$

$\Rightarrow$ $\boxed{\text{Conclusion}}$: 4936.7 to 5003.3 is a 95% confidence interval for the mean compressive strength.

4. 4. A fire insurance company felt that the mean distance from a home to the nearest fire department in a suburb of Chicago was at least 4.7 mi. It set its fire insurance rates accordingly. Members of the community set out to show that the mean distance was less than 4.7 mi. This, they felt, would convince the insurance company to lower its rates. They randomly identified 64 homes and measured the distance to the nearest fire department for

$< 4.7 mi$

$n = 64$

accordingly. Members of the community set out to show that the mean distance was less than 4.7 mi. This, they felt, would convince the insurance company to lower its rates. They randomly identified 64 homes and measured the distance to the nearest fire department for each. The resulting sample mean was 4.4. Assume $\sigma = 2.4$ mi. (known)

< 4.1 mi

$n = 64$

$\bar{x} = 4.4$

a. Does the sample show sufficient evidence to support the community's claim at the $\alpha = 0.05$ level of significance?

b. Find a 98% confidence interval for the mean distance from home to the nearest fire department.

(a) step 1: Set-up.

$\boxed{a}$ Parameter of concern: The mean distance from a home to the nearest fire department.

$\boxed{b}$ $H_0: \mu = 4.7$ ($\geq$)

$H_a: \mu < 4.7$

step 2: The Hypothesis Test Criteria.

$\boxed{a}$ normality assumed / check assumption

$n = 64$ big enough, "CLT"

$\boxed{b}$ The test statistic z, with $\sigma = 2.4$ known.

$\boxed{c}$ Determine the level of significance: $\alpha = 0.05$.

step 3: The sample evidence.

$\boxed{a}$ The sample information: $\bar{x} = 4.4$, $n = 64$.

$\boxed{b}$ Calculate the value of test statistic

$$z^* = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{4.4 - 4.7}{\frac{2.4}{\sqrt{64}}} = -1.00$$

step 4: The probability Distribution.

① P-value approach

$P = P(z < z^*) = P(z < -1.00)$

$= P(z > 1.00)$

② Classical approach

$-z(\alpha) = -z(0.05)$

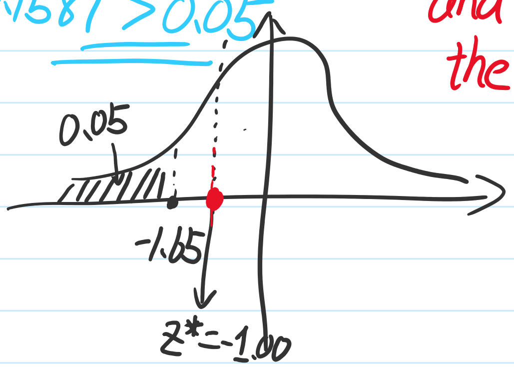
$= -1.65$

$= P(z > 1.00)$

$= 0.1587 > 0.05$

$= -1.65$

and $z^* = -1.00$ falls in the non-critical region.



Step 5: The Results:

$\boxed{a}$ Decision: Fail to reject H_0.

$\boxed{b}$ Conclusion: At the 0.05 level of significance, the sample does not provide sufficient evidence to support Community's claim.

#.

(b) $\bar{x} - z(\frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}$ to $\bar{x} + z(\frac{\alpha}{2})\frac{\sigma}{\sqrt{n}}$.

$n = 64$, $\bar{x} = 4.4$, $\sigma = 2.4$, $\alpha = 0.02$.

✓Confidence coefficient: $z(\frac{\alpha}{2}) = z(0.01) = 2.33$.

✓Maximum error: $E = z(\frac{\alpha}{2})\frac{\sigma}{\sqrt{n}} = 2.33\,\frac{2.4}{\sqrt{64}} = 0.699$

A 98% confidence interval for μ is

$4.4 - 0.699$ to $4.4 + 0.699$

i.e. 3.701 to 5.099.

5. A dog-food manufacturer sells "50-lb" bags of dog food. It is known that the standard deviation of weights for all bags of this brand is 0.84 lb. Suppose you randomly select 25 bags and find that $\bar{x} = 50.19$ lb. Would you be inclined to believe that the actual mean weight, μ, of all "50-lb" bags of this dog food differs from the advertised weight of 50 lb? Perform your hypothesis test at the 5% significance level.

$\sigma = 0.84$

$\alpha = 0.05$

Pf: Step I: a. parameter of concern: weight of dog food.

b. H_0: $\mu = 50$ lb

b. $H_0: \mu = 50\,lb$

$H_a: \mu \neq 50\,lb$

Step II: a. normality assumed, "CLT" with $n=25$.

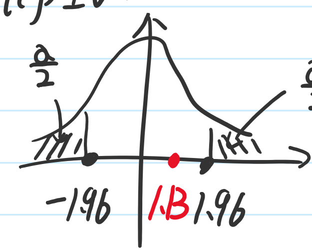
b. σ known $=0.84$ c. $\alpha = 0.05$.

Step III: a. sample information: $n=25$, $\bar{x} = 50.19\,lb$.

b. test statistic

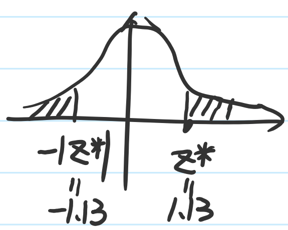
$$z^* = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{50.19-50}{\frac{0.84}{\sqrt{25}}} = 1.13.$$

Step IV:

① 

z^* is in the non-critical region.

② H_a contains $\neq$ (Two-tailed)



$$\text{P-value} = P(z < -|z^*|) + P(z > |z^*|)$$
$$= 2 \times 0.1292$$
$$= 0.25 > 0.05.$$

Step V: a. Fail to reject H_0

b. At the 0.05 level of significance, the sample does not provide sufficient evidence to believe that it differs from the advertised weight of 50 lb.

---

6. A normally distributed population is known to have a standard deviation of 5, but its mean is in question. It has been argued to be either $\mu = 80$ or $\mu = 90$, and the following hypothesis test has been devised to settle the argument. The null hypothesis, $H_0: \mu = 80$, will be tested using one randomly selected data and comparing it to the critical value 86. If the data is greater than or equal to 86, the null hypothesis will be rejected.

$\sigma = 5$

≥ 86

a. Find α, the probability of the type I error.

b. Find β, the probability of the type II error.

c. Suppose the argument was to be settled using a sample of size 4; find α and β.

Hypothesis test outcomes:

Decision	Null Hypothesis True	Null Hypothesis False
Fail to reject H_0	Type A correct decision	Type II error
Reject H_0	Type I error	Type B correct decision

Type A correct decision:

Null hypothesis true, decide in its favor.

Type B correct decision:

Null hypothesis false, decide in favor of alternative hypothesis.

Type I error: H_0

Null hypothesis true, decide in favor of alternative hypothesis.

Type II error:

Null hypothesis false, decide in favor of null hypothesis. 17

H_0

(a). $\alpha = P$(rejecting H_0 when H_0 is true)

$= P(x \geq 86 \mid \mu = 80)$

$= P\left(\frac{x-\mu}{\sigma} \geq \frac{86-80}{5}\right)$

$= P(z \geq 1.2)$

$= 0.1151$

(b) $\beta = P$(accepting H_0 when H_0 is false)

$= P(x < 86 \mid \mu = 90)$

$= P\left(\frac{x-\mu}{\sigma} < \frac{86-90}{5}\right)$

$= P(z < -0.8)$

$= 0.2119.$

(C) $n = 4$, $\bar{x}$

$\sigma = 5.$

$\alpha = P$(rejecting H_0 when the H_0 is true)

$= P(\bar{x} \geq 86 \mid \mu = 80)$

$= P\left(\frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}} = (\frac{\sigma}{\sqrt{n}})} \geq \frac{86-80}{(\frac{5}{\sqrt{4}})}\right)$

$= P(z > 2.4)$

$= 0.0082.$

$\beta = P$(accepting H_0 when H_0 is false)

$= P(\bar{x} < 86 \mid \mu = 90)$

$= P\left(\frac{\bar{x}-\mu_{\bar{x}}}{\sigma_{\bar{x}} = (\frac{\sigma}{\sqrt{n}})} < \frac{86-90}{\frac{5}{\sqrt{4}}}\right)$

$= P(z < -1.6)$

$= 0.0548.$

7. A major manufacturing firm producing PCB (a dangerous substance) for electrical insulation discharges small amounts from the plant. The company management, attempting to control the PCB in its discharge, has given instructions to halt production if the mean amount of PCB in the effluent exceeds 3 parts per million (ppm). From a random sample of 50 water specimens, the sample mean is 3.1 ppm. Assume that $\sigma = 0.5$ ppm and use $\alpha = 0.01$.

$\mu > 3$ ppm

$n = 50$

a. Do the statistics provide sufficient evidence to halt the production process?

b. Calculate β, the type II error, for the test described in part ~~(b)~~ (a) if the true mean is $\mu = 3.2$ ppm.

c. What will be the minimum sample size of water specimens if the value of β in part ~~(c)~~ (b) is restricted to be less than 0.15 while the value of α remains unchanged?

Pf: (a) Parameter of Concern: mean amount of PCB

P5: (a) Parameter of Concern: mean amount of PCB

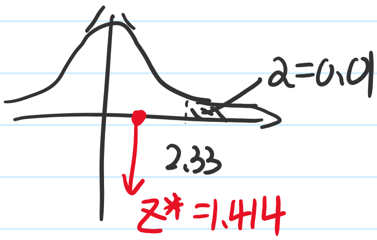
$H_0: \mu \le 3\text{ppm}$

$H_a: \mu > 3\text{ppm}.$

$\bar{x} = 3.1\text{ppm},\ \sigma = 0.5\text{ppm},\ \alpha = 0.01,\ n = 50.$

$$z^* = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}} = \frac{3.1-3}{\frac{0.5}{\sqrt{50}}} = 1.414$$

$z\text{-critical} = z(\alpha) = z(0.01) = 2.33.$

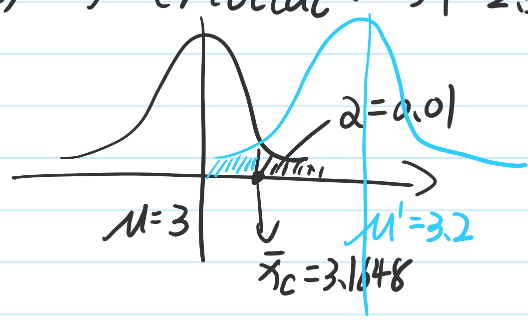


z* is NOT in the critical region.

$\Rightarrow$ Fail to reject H_0.

Therefore, the statistic do NOT provide sufficient evidence to halt the production process, at the 0.01 level of significance.

(b) $\bar{x}$-critical: $3 + 2.33 \times \frac{0.5}{\sqrt{50}} = 3.1648$



Non-critical region

$$\beta = P(\bar{x} < 3.1648 \mid \mu' = 3.2)$$

$$= P\left(z < \frac{3.1648-3.2}{\frac{0.5}{\sqrt{50}}}\right)$$

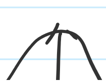
$$= P(z < -0.4984)$$

$$= 0.3085$$

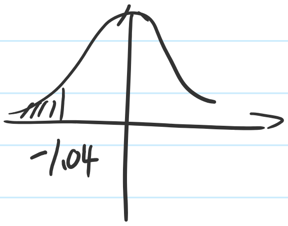
(c) $\bar{x}_c'$ be new $\bar{x}$-critical point "n" unknown

Then, $\bar{x}_c' = 3 + 2.33 \times \frac{0.5}{\sqrt{n}}$

Since $\beta = P\left(z < \frac{\bar{x}_c' - 3.2}{\frac{0.5}{\sqrt{n}}}\right) < 0.15$, $-z(0.15) = -1.04.$



$P\left(Z < \frac{\quad}{\frac{0.5}{\sqrt{n}}}\right) < 0.15$, $-z(0.15) = -1.04$.



$$\frac{\bar{x}_c' - 3.2}{\frac{0.5}{\sqrt{n}}} < -1.04$$

$$\Rightarrow 3 + 2.33 \times \frac{0.5}{\sqrt{n}} - 3.2 < -1.04 \times \frac{0.5}{\sqrt{n}}$$

$$\Rightarrow n > 70.98 \Rightarrow \min n = 71. \quad \#$$

8. The breaking strength of a fiber used in manufacturing certain cloth is required to be not less than 130 psi. A random sample of eight specimens is tested and the breaking strengths (in psi) are shown below: $n=8$

125.4 134.6 122.8 132.7 120.9 121.1 126.2 127.5

Assume that the breaking strength of that kind of fiber has a normal distribution with population standard deviation of 6 psi. $\sigma = 6$

a. At the 0.01 level of significance, do the data provide sufficient evidence to conclude that the mean breaking strength of the fiber is less than 130 psi? $\mu < 130$

b. Suppose the probability of Type II error in the hypothesis test of part (a) is estimated to be 0.12. Calculate the actual mean breaking strength of the fiber if the null hypothesis is false.

Pf (a): parameter of Concern: mean breaking strength of fiber.

$H_0: \mu \geq 130$ psi

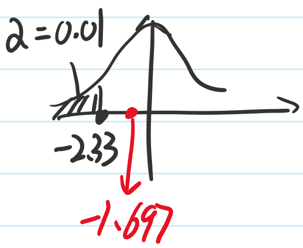
$H_a: \mu < 130$ psi.

Sample information: $\sigma = 6$ psi, $\alpha = 0.01$, $\bar{x} = 126.4$ psi, $n = 8$.

$$z^* = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{126.4 - 130}{\frac{6}{\sqrt{8}}} = -1.697$$

① Classical approach:

z-critical $= -z(0.01) = -2.33$



z^* lies in non-critical region

② P-value approach: P-value $= P(Z < -1.697) = 0.0446 > \alpha = 0.01$.

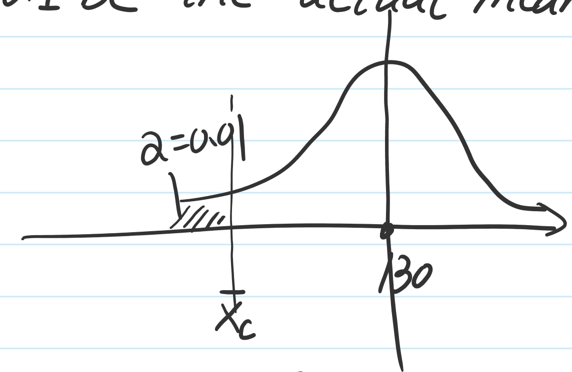
$\Rightarrow$ Fail to reject H_0

P-value $= P(Z < \ldots) = 0.0446 > \alpha = 0.01$.

$\Rightarrow$ Fail to reject H_0

There is NOT sufficient evidence to conclude that $\mu < 130$ psi at the 0.01 level of significance.

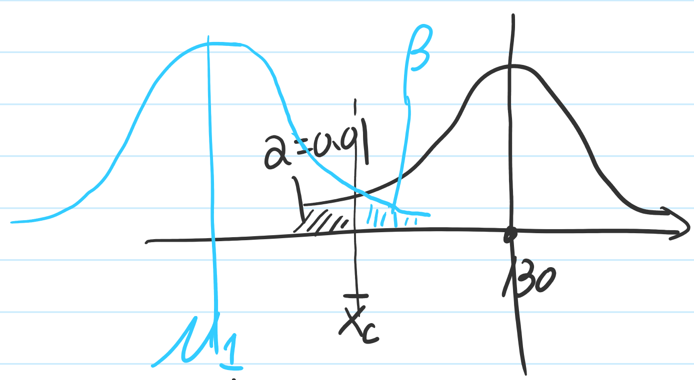
(b) Let μ_1 be the actual mean breaking strength of fiber,



If H_0 is true, $\dfrac{\bar{x}_c - 130}{\frac{6}{\sqrt{8}}} = -z(0.01)$

$= -2.33$

$\Rightarrow \bar{x}_c = 125.05732$



If H_0 is false, $\dfrac{\bar{x}_c - \mu_1}{\frac{6}{\sqrt{8}}} = \overset{\beta}{z(0.12)} = 1.175$.

$\Rightarrow \mu_1 = 125.05732 - 1.175 \times \frac{6}{\sqrt{8}}$

$= 122.5648$ psi #.

Take-home Assignment 1(feedback)

Friday, October 18, 2019 1:41 PM

1.

Question 1 [20 marks]

The probabilities that John will choose Physics, Chemistry and Biology in the summer semester are 0.4, 0.7 and 0.6, respectively. If John has decided to choose Physics, the probabilities of choosing Chemistry and Biology are increased to 0.8 and 0.75, respectively. On the other hand, if he has decided not to choose Biology, the probability of choosing Chemistry becomes 0.6. If John has decided to choose Biology, the probability of choosing Chemistry but not Physics becomes 0.35.

$P(P)=0.4 \quad P(B)=0.6$

$P(C)=0.7$

$P(C|P)=0.8 \quad P(B|P)=0.75$

$P(C|\bar{B})=0.6$

$P(C\cap\bar{P}|B)=0.35$

(a) What is the probability that John will choose all three courses? [12 marks]

(b) What is the probability that John will not choose any course in the summer semester? [5 marks]

(c) If John has decided to choose Chemistry, what is the probability that John will choose exactly two courses in the summer semester? [3 marks]

$$P(C|P)=\frac{P(C\cap P)}{P(P)} \Rightarrow P(C\cap P)=P(P)\cdot P(C|P)=0.4\times0.8=0.32$$

$$P(B|P)=\frac{P(B\cap P)}{P(P)} \Rightarrow P(B\cap P)=P(P)\cdot P(B|P)=0.4\times0.75=0.3.$$

$$P(C|\bar{B})=\frac{P(C\cap\bar{B})}{P(\bar{B})} \Rightarrow P(C\cap\bar{B})=P(\bar{B})\cdot P(C|\bar{B})=(1-0.6)\times0.6=0.24.$$

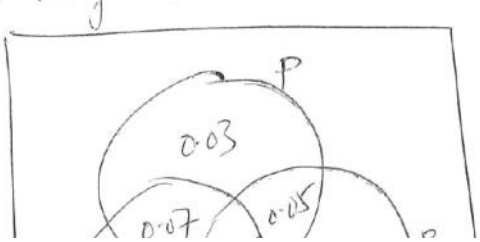
$$\triangle\, P(C\cap B)=P(C)-P(C\cap\bar{B})=0.7-0.24=0.46.$$

$$P(C\cap\bar{P}|B)=\frac{P(C\cap\bar{P}\cap B)}{P(B)} \Rightarrow P(C\cap\bar{P}\cap B)=P(C\cap\bar{P}|B)\cdot P(B)$$
$$=0.6\times0.35=0.21$$

(a)

$$\triangle\, P(C\cap P\cap B)=P(C\cap B)-P(C\cap B\cap\bar{P})=0.46-0.21=0.25.$$

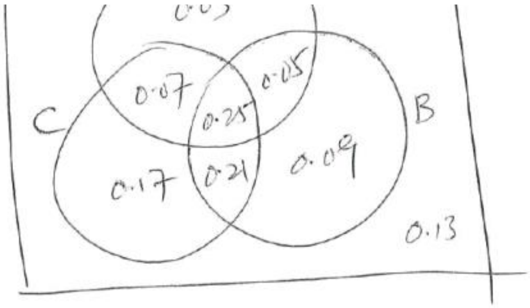
Venn Diagram



(b) $P(P \text{ or } C \text{ or } B)$

$=P(P)+P(C)+P(B)-P(P\cap C)$

$-P(P\cap B)-P(C\cap B)+P(P\cap C\cap B)$



$-P(P\cap B)-P(C\cap B)+P(P\cap C\cap B)$

$=0.4+0.7+0.6-0.32-0.3-0.46+0.25$

$=0.87$

$P(\overline{P\cap C\cap B})=1-0.87=0.13.$

(C) $P(\text{exactly two course}\mid C)=\dfrac{0.07+0.21}{P(C)}=\dfrac{0.28}{0.7}=0.4.$ #

Question 2 [20 marks]

2. The numbers of different types of power stations being operated in four countries are shown in the table.

	Coal	Nuclear	Hydroelectric	Total
China	38	10	68	116
Finland	2	2	x	4+x
Germany	32	23	7	62
Japan	4	21	63	88

(a) If a nuclear power station is selected at random, what is the probability that it was from Japan? [5 marks]

(a) $P(\text{Japan}\mid \text{Nuclear})$

$=\dfrac{21}{10+2+23+21}=0.375$

(b) If two power stations are selected from either China or Japan, what is the probability that none of them are coal-generated? [7 marks]

(b)

	Coal	Nuclear	Hydroelectric	Total
China or Japan	42	31	131	204.

$\dfrac{C^{2}_{162}}{C^{2}_{204}}$

$P(\text{None are coal}\mid \text{China or Japan})=\dfrac{31+131}{204}\times\dfrac{31+131-1}{204-1}=0.6298.$

(c) Let PC and PH be the events "Power station is from China" and "The power station is hydroelectric", respectively. How many hydroelectric power stations are there in Finland if PC and PH are independent events? [8 marks]

(c) $P(PC\mid PH)=P(PC)$

$\Rightarrow \dfrac{68}{68+x+7+63}=\dfrac{38+10+68}{76+56+138+x}$

$68+57+63$ $\qquad$ $76+56+138+x$

$\Rightarrow 68(270+x) = 116(138+x)$

$\Rightarrow 48x = 2352$

$\Rightarrow x = 49.$ #

3、 **Question 3** [20 marks]

In a production process, a total number of 100 items were produced by machines M_1, M_2 and M_3. Of those items being produced, 37 were made by machine M_1, 42 were made by M_2, and 21 were made by M_3. The three machines work independently, however, they do not work perfectly. From the experience, 5% of the items produced by M_1 are defective, 4% of the items produced by M_2 are defective, and 3% of the items produced by M_3 are defective.

$P(M_1) = 0.37$ $\quad$ $P(M_3) = 0.21$

$P(M_2) = 0.42$

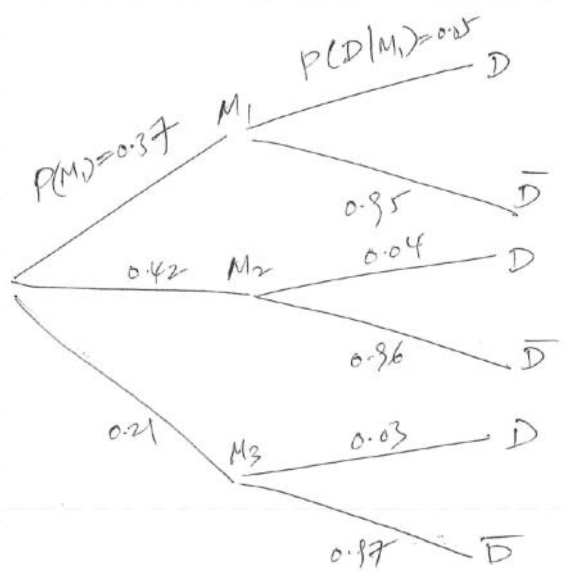
$P(D|M_1) = 0.05$

$P(D|M_2) = 0.04$

$P(D|M_3) = 0.03.$

(a) Given that a randomly selected item is non-defective ($\bar{D}$), what is the probability that it is produced by machine M_1? [8 marks]

(b) If two items which are not produced by machine M_3 are selected at random without replacement, what is the probability that at least one of them is non-defective? [12 marks]



(a) $P(M_1|\bar{D}) = \dfrac{P(M_1 \cap \bar{D})}{P(\bar{D})}$

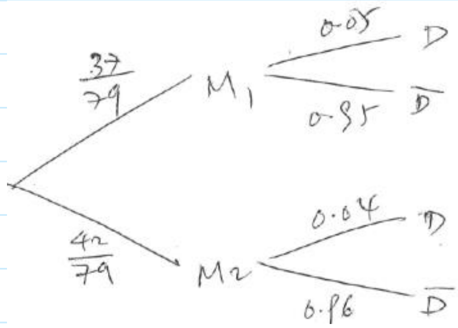
$\Rightarrow P(M_1 \cap \bar{D}) = 0.37 \times 0.95 = 0.3515$

$P(\bar{D}) = 0.37 \times 0.95 + 0.42 \times 0.96 + 0.21 \times 0.97$

$= 0.9584$

$\Rightarrow P(M_1|\bar{D}) = \dfrac{0.3515}{0.9584} = 0.36675$

(b) $P(\text{at least } 1\ \bar{D} \mid M_1 \text{ or } M_2) = 1 - P(\text{Both are } D \mid M_1 \text{ or } M_2)$



$$=1-\left(\frac{37}{79}\times 0.05\times\frac{36}{78}\times 0.05+\frac{42}{79}\times 0.04\times\frac{41}{78}\times 0.04+2\times\frac{37}{79}\times\frac{42}{78}\times 0.05\times 0.04\right)$$

$$=1-0.002$$

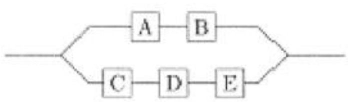
$$=0.998$$

#

4.

Question 4 [20 marks]

A system consists of five components in two branches as shown in the following diagram:



In other words, the system works if components A and B work or components C, D, and E work. Assume that the components fail independently with the following probabilities:

$$P(A \text{ fails}) = P(B \text{ fails}) = 0.1 \quad \text{and} \quad P(C \text{ fails}) = P(D \text{ fails}) = P(E \text{ fails}) = 0.2.$$

(a) What is the probability that the system works? [8 marks]

(b) Given that the system works, what is the probability that component A does not work? [6 marks]

(c) Given the system does not work, what is the probability that component A also does not work? [6 marks]

(a) $P(\text{system work})$

$=P(A\cap B \text{ or } C\cap D\cap E)$

$=P(A\cap B)+P(C\cap D\cap E)-P(A\cap B\cap C\cap D\cap E)$

$=0.9^2+0.8^3-0.9^2\times 0.8^3$

$=0.90728.$

(b) $P(\bar{A}\mid \text{system work})$

$$=\frac{P(\bar{A}\cap \text{system work})}{P(\text{system work})}=\frac{P(\bar{A}\cap C\cap D\cap E)}{P(\text{system work})}=\frac{0.1\times 0.8^3}{0.90728}=0.05643$$

(c) $$P(\bar{A}\mid \text{system fail})=\frac{P(\bar{A}\cap \text{system fail})}{P(\text{system fail})}$$

$$=\frac{P(\bar{A})-P(\bar{A}\cap \text{system work})}{P(\text{system fail})}$$ ✓

$$=\frac{0.1-0.1\times 0.8^3}{1-0.90728}=0.52632.$$

#

5.

Question 5 [20 marks]

(a) A fair coin is tossed until a head is obtained. What is the probability that the number of tosses required is an odd number? [10 marks]

5、 **Question 5** [20 marks]

(a) A fair coin is tossed until a head is obtained. What is the probability that the number of tosses required is an odd number? [10 marks]

(b) A fair die is thrown seven times. Find the probability that the outcome contains one '1', two '2', three '3', but no '4'? [10 marks]

(a) E_k = "k^{th} toss is head", $\overline{E_k}$ = "k^{th} toss is tail"

$$P(k \text{ is odd}) = P(E_1) + P(\overline{E_1}\,\overline{E_2}E_3) + \cdots + P(\overline{E_1}\,\overline{E_2}\cdots\overline{E_{k-1}}E_k)$$

$$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \cdots \qquad k \text{ is odd}$$

common ratio: $r = \frac{1}{2^2}$

$$= \frac{\frac{1}{2}\left[1-\left(\frac{1}{2^2}\right)^k\right]}{1-\frac{1}{2^2}} = \frac{2}{3}.$$

(b) $\underline{1}$ $\underline{2}$ $\underline{2}$ $\underline{3}$ $\underline{3}$ $\underline{3}$ $\underline{*}$ NOT 1, 2, 3, 4 Only 5, 6

(I) $P = \dfrac{\binom{6}{3}\binom{3}{2}\binom{1}{1}\cdot 7\cdot 2}{6^7} = 0.003$

(II) $P(\text{above}) = \frac{1}{6^6} \times \frac{2}{6}$

$$P = \binom{7}{1}\cdot\binom{6}{2}\binom{4}{3}\cdot\left(\frac{1}{6^6}\times\frac{2}{6}\right) = 0.003.$$